POLSKA AKADEMIA UMIEJĘTNOŚCI
INSTYTUT EKONOMICZNY

STUDIA EKONOMICZNE

ECONOMIC STUDIES

III

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SKŁAD GŁÓWNY W KSIĘGARNIACH GEBETHNERA I WOLFFA
WARSZAWA—KRAKÓW—ŁÓDŹ—POZNAŃ—WILNO—ZAKOPANE

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REDAKCJA (EDITORS): PROF. DR ADAM HEYDEL
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SPIS TREŚCI

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DOREEN WARRINER

LAND REFORM AND THE PROBLEM OF CAPITALISATION [1]

The view that the agrarian reform in Europe represents a process of decapitalisation is very generally expressed in the literature of the subject: it is, for example, the theory held by Sering, by S. Schmidt [2], and most explicitly stated by Oberländer [3], who asserts that the reform destroyed considerable capital values, and at the same time increased the demand for capital from the new farms without satisfying it. This assertion seems to indicate such a profound misunderstanding of the results of the reform that it deserves examination: and it is the object of this paper to suggest that the argument is based on theoretical assumptions about the rate of investment and the direction of economic progress which may be appropriate in other conditions, but which do not appear to be justified in Eastern Europe.

None of these writers define what they mean by the term decapitalisation, and are not aware of any difficulty inherent in discussing the effect of the measure on the amount of capital.

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[1] Miss Doreen Warriner, lecturer at University College, London, is investigating agrarian conditions in Central, and East European countries as a Rockefeller fellow.

Miss Warriner deals in this paper with the fundamental economic problems of agrarian conditions.

The editors of the *Studja Ekonomiczne* are publishing this study in the hope that it may evoke a discussion, and throw light on one of the most important problems of the economy of Central and Eastern Europe.

[2] *Berichte über Landwirtschaft, Die Landwirtschaft Polens*, Berlin 1928.

[3] *Die agrarische Uebervölkerung Polens*, Berlin 1935.

It seems possible that two meanings are intended; either a general disinvestment in the Hayekian sense of a process of capital consumption, a reduction of savings below the amount sufficient to maintain the existing capital goods in repair — or in the sense of a misdirected investment, which causes a diversion of savings into channels where the rate of interest is lower than it would be in other branches of production. Both processes result in a destruction of future income.

Thus the land reform might be held to influence capital investment in two ways, according to the sense in which the term decapitalisation is used. First, it could affect the amount of savings directly, if the division of units affects the use of existing capital equipment. Cutting up the big estates is likely to reduce the efficiency of labour, if capital goods exist in the form of large complexes of buildings, power plant and machinery, which cannot be operated successfully if divided among several units and may even become entirely useless. Complete subdivision would cause a catastrophic decline in output per head, because it would cause a return to less capitalistic methods of production. Second, it could affect the direction of investment by favouring types of farming enterprises which use more labour and more capital per acre than the large farm. It would therefore change the demand for factors of production.

To use the first method of approach, and discuss the effects of the reform simply as a division of large units, it is necessary to refer to some principle which determines the scale of farming operations: if the existence of large and small farms side by side is simply due to the survival of an obsolescent social order, the scale of farming operations is, from the economic standpoint, indeterminate, and the agrarian reform would have no economic significance at all. Unfortunately, when we attempt to give this idea of efficient farm sizes any precise meaning, it is difficult to avoid raking over the ashes of recent controversies on the theory of partial equilibrium. In this discussion, the conception of an optimum firm has been related to that of a supply curve, i. e. a curve which is derived from assuming that one factor is constant and that others are available in perfectly elastic supply to the entrepreneur: a curve of this sort must have a point where the cost per unit of output is

lowest, when a certain relationship exists between the fixed factor and the amounts of the others. From the entrepreneur's standpoint this means that he regards certain costs as fixed independently of the amount produced and others as varying directly with it; but there is no reason to regard any single element as the fixed one: it may be his own management, capital goods, a piece of land, so that the idea has no practical significance. The air of relevance which it at one time acquired through the Cambridge convention of defining the short period as that in which the capital equipment cannot be changed, has been dispersed by the discussion on the nature of the long period optimum [1]. From the standpoint of general equilibrium theory, there can be no sense in regarding any factor as more fixed in supply than the others, though possibly it may be legitimate to use the conception of a producing unit in the sense of a ratio between indivisible and divisible units of factors, and therefore to admit the existence of optimum sizes in the sense of a fixed ratio which must exist between the units of the indivisible factors and the others.

Such units as these must exist in all branches of production: but it is difficult to decide which of them are to be regarded as criteria of efficient farm sizes. The only method seems to be to ask what in fact do farmers regard as their main fixed cost. Whereas in England it would be roughly accurate to regard the machine capital as an indivisible unit which determines at least the minimum area for specific types of farming, in Eastern Europe the level of mechanisation on large and small farms alike is so low that this standard has little meaning. To use the old traditional unit, the basis of many land tenure systems, the ploughing team, would still be a good measure for the smaller farms: and on this standard the majority of farms under five hectares are too small in area, and carry an excessive number of draught animals in relation to their size. For the large farm, on the other hand, this standard is inapplicable: they are merely agglomerations of ploughing units, using better and heavier machinery, though rarely any motor power. In their case the only basis for calculating optimum areas

[1] cf. N. Kaldor, *Equilibrium of the Firm*. Economic Journal, March 1934.

would appear to be the buildings: and in fact it appears to be a working principle among big farmers to regard the building capital as their main fixed cost. If the farm is sub-divided it is mainly the buildings capital which becomes excessive.

Of course we cannot suppose that two optimum scales of output exist simultaneously for the same type of production: if the big farms economy of building capital really indicates a cost advantage, the smaller farms would combine. But since there is almost no upward movement from one class to the other, the existence of the two types of farm side by side must be attributed in the main to institutional factors, i. e. to the circumstances of peasant emancipation, and — to a lesser extent — to the fact that, in view of the surviving social cleavage, the peasant supplies his labour and capital at lower prices to his own enterprise — or, in other words, the factors have different prices in the different classes. Thus it is legitimate to suppose that within each of the range of farm sizes there are areas which, in connection with the existing capital equipment, will produce at lowest cost per unit. The smaller peasant farms, even making allowance for the lower return which they expect on their investment, carry an excessive amount of capital invested in horses. In consequence of their high costs, transfer of land from large farms to small farms should continue until the reduction of costs on the peasant farms is offset by the rise in building costs per unit of output on large farms.

Using this method of approach, assuming that the areas of the big farms more or less approximate to the optimum for their buildings and that the areas of small farms are very frequently too small in relation to their livestock capital, it seems that land reform legislation is justifiable in the rather restricted sense of a credit operation which would facilitate the transfer of land and allow the peasant to increase the efficiency of his labour before he has finished the purchase of additional land. This transfer would reduce the areas of big farms, which are presumed to be optimum areas, and so would raise their unit costs of production: the question is simply whether the reform has caused a wastage of the building capital which is not compensated for by the increased efficiency of the working capital of the peasant farm. This is a question

which is not suited to statistical treatment, since it depends simply on the quantitative significance which we attribute to the excess capacity. Reasons can be adduced for thinking that there has not been any reduction of efficiency from the breaking up of units so far as Poland is concerned. In the provinces formerly included in Prussia, where large heavily capitalised units farm very intensively on poor soil, very little settlement has been done: the land reform has taken the form of parcellisation — sales of small pieces of land to adjacent farms: there has been nothing resembling the complete division of estates and conversion of buildings to peasant houses, which has taken place under the settlement legislation in Eastern Germany. The industrial farms, that is, those which own a distillery or other industrial plant, have been excluded from the legislation. On the basis of the facts, it seems difficult to believe that there has been any decline in productivity resulting from the subdivision — from the sub-division alone, that is to say.

However, it is quite evident that the optimum firm method of approach does not touch the real problem. To use it we have to suppose that efficient operation depends simply on utilising to its full capacity capital equipment which, it is assumed, is to be maintained intact. Increased or decreased saving, changes in the prices of the factors or technical progress, may make it uneconomic to maintain this equipment at all. The important aspect of the reform is not that it causes a division of the units but that it promotes a change in the use of the factors of production. This it does in two ways: it facilitates investment in types of agricultural enterprise which use more labour and capital in relation to land, and it encourages investment in agriculture as contrasted with industry — that is, in branches of production which in general use less capital in relation to labour. To what extent this tendency represents a misdirected investment we cannot decide unless we have in mind some general presumption about the way in which the prices of factors of production are changing in relation to each other.

To make this presumption for Western Europe is very simple, so simple that it is hardly ever made: "the only situation of which we have any knowledge in capitalistic society is one

in which total investment is growing at a fairly rapid rate"[1]. That is to say, it is always assumed that there is economic progress, and that labour rises in price. Consequently an agricultural policy which would cause a division of farms, and therefore in an increased use of labour, would be an uneconomic policy from a long period standpoint. Without attempting to show any causal connection between the phenomena, we can describe the course of economic development in the past fifty years as a rise in the income level, accompanied by a growth in total population, increasing productivity per head and investment preponderantly in new means of transport to bring food supplies to the European market. To all entrepreneurs these changes appear in the form of a rise in the price of labour: in industry entrepreneurs experience an increased demand for their products and in agriculture — apart from certain special branches — they experience a decreased demand. In consequence the main features of European agriculture in the last fifty years have been a movement of labour into industry and an increased substitution of capital for labour, both aspects of a rapid rate of capital accumulation. In spite of the striking differences in agrarian structure and the status of landworkers which loom rather large to English observers, there is no essential difference between England and Western Europe as regards the basic economic conditions of their agriculture: in both the rural exodus, increasing output per man, rising labour costs, and falling food prices are features which in different degrees can be taken for granted (though it seems possible that in England in the future a decline in capital invested in agriculture will occur and cause a contraction of the total output).

In spite of the impact of these forces, there is no sign of any important change in the scale of farming operations or the agrarian structure: the relation of the different size groups to each other remains more or less unchanged, with a clear tendency in many countries towards a relative increase of the

[1] Knight, *Capital Time and the Interest Rate*, Economica, August 1934. Hayek's treatment also takes it for granted that as a long run phenomenon capital consumption will not attain a scale which will exceed the simultaneous formation of new capital (*Kapitalaufzehrung*, Weltwirtschaftliches Archiv, Juli 1932).

medium sizes farms as compared with the very large and the very small. But although the structure has not undergone much apparent change, there certainly has been a change in the production co-efficients in that methods requiring the use of more capital per man have been introduced, and, to a lesser extent, the use of more land per man.

In Eastern Europe, none of these features are apparent, and the same general presumption cannot be made: industrialisation proceeds very slowly, most of the increase of population remains on the land, and output per man increases very little, if at all. There is evidence to suggest that in Poland output per man in fact decreases: economic development in a long period sense is going in a backward direction [1]. A very low level of income per head, and much hidden under-employment in agriculture exist simultaneously with a very high density of agricultural population; and it is generally believed that there is a causal connection between the low standard of living and the density of population — in other words, that over-population causes under-employment and low output per man. This thesis has been put forward in a recently published work by J. Poniatowski [2], and by Oberländer in the study to which reference has been made. If true, it implies, a general disinvestment; prices of the factors of production are changing in an opposite direction to that of Western Europe: wages are falling, the price of land and capital goods rising. The cost of maintaining the existing capital goods is also rising, and the rate of new investment is therefore slower. In other words, savings per head do not suffice to make additions to the existing capital goods in proportion to the growth of population. Under these circumstances the ratio of the factors of production on the big estates cannot be maintained. In the absence of a faster rate of savings, additional units of output must be produced with increased use of labour in relation to capital.

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[1] Certainly as far as Galicia is concerned there seems to be no doubt that output per head is declining: the fact of a general retrograde development in Congress Poland seems difficult to establish in view of the calculations made by W. Liskiewicz, *Problemat gospodarczego wzrostu w rolnictwie polskiem w epoce 1822—1931*, Ekonomista II 1933.

[2] *Przeludnienie wsi i rolnictwa*, Warszawa 1936.

In considering the effects of the reform it is necessary to decide whether this theory of the course of economic development in Poland is really a true explanation, since obviously it changes the approach to the question of capitalisation.

On the basis of the observed facts, two explanations seem possible. One is that there is a rate of capital accumulation which is sufficient to keep output per man stable, but not sufficient to increase it; the other is that the rate of capital accumulation has not kept pace with the growth of population. Certainly the most optimistic view possible is that the standard of living has not fallen since the pre-war period. The former explanation implies that there has been no new industrial development (and no technical change in agriculture): as population increases, the amount of capital increases sufficiently to maintain income per head, and demand for agricultural products increases also, so that the additional population will be employed on the land and in industry in the same proportion as before. Keeping to the assumption that the peasant prices his labour and capital cheaper when used on his own holding, the increase in supply of labour and capital will represent an increased demand for land and the landowners will sell to the small farms.

The break-up of the latifundia has been a process of capital investment of this kind. For the past sixty years the big estates have been subject to a process of attrition, which was always accelerated when savings increased in years of prosperity; in Galicia it was mainly the savings of emigrants which bought up the big estates. The process seems to have affected the agrarian structure in two ways: in Congress Poland causing an increase in the area of the medium sized peasant farms at the expense of the big estates [1], in Galicia a sub-division of all sizes of holdings, both large estates and medium sized peasant farms [2]. This process may by regarded as inevitable, in the

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[1] See Władysław Grabski, *Materjały w sprawie włościańskiej*, I. Rozdział VI.

[2] A study of the history of twenty Galician villages in the period 1787 to 1931 shows a continuous decline in the average size of peasant farms; Styś, *Rozdrabnianie gruntów chłopskich w byłym zaborze austrjackim od roku 1787 do 1931*, Lwów 1934.

absence of a rate of savings sufficient to permit industrialisation.

Under these circumstances it is clear that the land reform only accelerates slightly a process which in any event would have occurred. In Galicia it has not even caused an acceleration of the rate of subdivision: the amount of land divided up in the ten years following the reform is less than it was the last ten years before the war. To regard the reform as if it were operating under the same conditions as those which exist in Western Europe is to miss its significance entirely: seen in its true perspective, it does not run counter to the direction which investment would normally take.

In these conditions the effect of the land reform legislation is to cause a transfer of assets from one group of producers to another, which will use the land in conjunction with a different combination of factors of production, by means of a state financed credit operation at an earlier date than would occur otherwise [1]. What is the significance of this? Obviously none, so far as the amount of capital is concerned, if at the later date the price of the asset transferred will be the same as it is at the earlier date: in this case the estate owner simply exchanges a claim to income from the land to claim to income from a mortgage or some form of land bond. If at both dates the price of the asset land is equal to its capitalised yield, it will be the same at both dates. It is immaterial for what purpose the credits are given: if a peasant gets credits at a lower rate of interest for the purchase of land he will invest a large proportion of his own savings in other assets, livestock and buildings, than he otherwise would do; if he received special credits for the purchase of these assets he would use his own savings for the purchase of land — any extension of agricultural credit for whatever purpose will cause a transfer of property. The transfer of the asset is only significant if between the two dates there

[1] In fact the reform has another important aspect: it secures a greater degree of homogeneity in the supply of land by fixing each year ahead a certain area to be divided in each district. To some extent, therefore, it causes a transfer of land which might not be sold at all, and therefore the terms for the landowner are less favourable than they would be: on the other hand in other districts the land reform enables the land-owner to sell on more favourable terms than he otherwise would.

is a change in the price of the asset. In the other case, when income per head is falling, this change must of course occur.

This explanation, however, would not be accepted by Poniatowski or Oberländer. Their theory implies that a decline in output per head is occurring, and many Polish economists would share the view that a retrograde development is in progress. But a decline in output per head may indicate a general and long period phenomenon, or it may be a short period due to losses of sources of income, territorial re-arrangements and other extraneous causes. To determine the effect of the land reform as a credit operation it is essential to decide which of these processes is occurring.

Poniatowski's theory is that it is a long period phenomenon due to the growth of population, and not to the extraneous causes, though these may intensify the effects of over-population. Hence it is necessary to see whether this view has any theoretical foundation, since there is not any prima facie reason why a rapid rate of population growth should affect adversely the accumulation of capital or the volume of employment: it might equally well be supposed to increase capital accumulation by raising the marginal productivity of investments.

The main difficulty is that the theory of over-population, as treated by these two writers, cannot be reconciled with any current theory of investment. To them both, the phenomena of over-population is a state which is measurable, i. e. is indicated by the existence of a number of unemployed or partially employed workers on the land. To estimate the number of these people both writers make a calculation of the number of workers who would be employed on the land under conditions of full employment. This figure Oberländer obtains by calculating the number of days work necessary to work an acre of land, and therefore the number of acres necessary to give full employment (reckoning some unemployment in winter as inevitable) to a peasant family. In order to make allowance for the greater intensity of cultivation in the Western provinces of Poland and in Galicia as compared with the East and Centre he uses a different standard for each province. On the basis of these calculations he arrives at the staggering re-

sult that in the year 1931 4·5 five million producers (42% of the total of those actively engaged) were superfluous. Poniatowski makes use of a more refined method, making a more exact allowance for the different degrees of intensity by calculating the area of Poland in weighted hectares, allowing a higher value to arable land and a low value to forests and pastures. The result of this method is to give a much lower estimate of the surplus population, 3·7 million for 1931. Neither of these writers is considering the phenomenon purely as a ratio between population and the area of land: obviously the result of the calculation depends entirely on the way in which the capital invested in agriculture is estimated.

The estimate, however made, must really mean that if the number of people were reduced by 30 or 40% the remainder could produce the same output and would have the same amount of capital to utilise, or, to put the same thing another way round, that the population has increased by 30 or 40% without any increase in investment, and therefore with a declining rate of saving per head. In itself, great density of agricultural population shows nothing: simply on this standard there are other areas of Europe which could be considered as much over-populated as Poland. The figure only has significance if it assumed that the increase has occurred without a proportionate increase in investment and that the existence of unemployment connected with the rate of population growth.

If general retrograde development, therefore is supposed to set in as a result of over-population this can only mean that, setting aside all extraneous causes, the destruction of the war, loss of confidence, territorial re-arrangements — all of which have sufficient magnitude to account for the declining rate of investment — the rate of growth of population somehow prevents capital accumulation from proceeding at the same rate. But the nature of this effect is differently explained. Oberländer seems to take it for granted on straightforward Malthusian lines that there is a decline in the physical yield of the soil: he defines the state of over-population by reference to an optimum, using a definition very characteristic of German economic thinking: it is "a ratio between a number of people and the *Nahrungsspielraum* at which the former is large enough to exploit with

their labour all the gifts of nature which are available under given economic and technical conditions, in such a way that the services of labour and the area of the *Nahrungsspielraum* produce the largest possible output under the given conditions". The idea behind this, if there is one, seems to be that the *Nahrungsspielraum* is an indefinable but fixed basis, which under indefinable conditions cannot be increased. Logically, of course, it is defensible, but fails to explain what the circumstances are in which the services of labour are unable to increase the supply of capital i. e. why and at what point the basis has to be taken as fixed, and thus assume the very point which it is desired to explain. Obviously if the basis is fixed any increase of population beyond any point must cause a decline in income per head.

This simple Malthusian type of explanation is not advanced by Poniatowski, who defines over-population in the only sense which seems permissible, i. e. that used by Wicksell, according to which over-population is presumed to exist if the rate of increase does not permit sufficient capital accumulation to maintain the output per head.

But what is the causal connection? Any theory of investment must be based on the possibility of comparing the value of the permanent future income to be obtained from the investment, with the value of present income which is invested, and this assumes some sort of time interval. This interval may be regarded as itself a determining factor as it is in Hayek's theory, or it may be regarded merely as one aspect of any investment, as it is by Knight: but it must exist. Population growth can be supposed to change its rate during this interval and must therefore cause changes both in the supply of the factors of production (and therefore in the cost of maintaining the capital) and in the prices of consumers goods, owing to the changed ratio of consumers to producers. If, at later date, the cost of maintaining the capital is higher because the propensity to consume is greater owing to the changed age composition of the population, the value of the additional income from the investment must be higher also. The amount of the investment which exists at the later date is determined by comparison of the yield at the later date with cost at an earlier date; therefore

it seems necessary to suppose that the change can be estimated — not in the sense that any individual investor will know exactly how population growth will affect his sort of investment, but in the sense that the general direction can be known.

The view that a change in the rate of population growth can influence the rate of investment seems to arise from a belief that the cost of saving rises simultaneously with a decline in the value (not the physical quantity) of the yield of an investment. This, of course, cannot be the case unless we assume the existence of some law of diminishing return on capital investment in general. On this point there is a sharp divergence between the two exponents of current theory. From the standpoint of Knight there can be no possibility of diminishing returns from capital investment, because investment implies a compound accumulation of new income yielding capacities [1]; the return from each investment increases the amount of factors available for further investment [2]. In the Hayek theory diminishing returns must be considered as a theoretical possibility, because the length of the investment period is something which operates independently to regulate the value of the yield of the investment, but from a recent article it is clear that diminishing returns are to be regarded as a theoretical possibility only [3]. Unless we retain the idea of a period of production, it

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[1] cf. Carlson, *On the Notion of Equilibrium in Interest Theory,* Economic Studies I, Kraków.

[2] Knight, *Economica,* August 1934, Capital, Time and the Interest Rate: "The heart of a correct theory of interest is the fact, corresponding more or less to infinite "elasticity of demand for capital" that the investment market is capable of absorbing savings at the maximum rate at which they are forthcoming, with only a very gradual decline of the rate of return through time, other things equal, and the further fact, that changes which do occur in the "other things" (partly in consequence of the growth of capital, but effects in a "historical" not an economic sense) actually prevent any general decline".

It follows that the cost of maintenance of capital must be treated as negligible, as also the existence of any agencies not subject to maintenance charge. (cf. Quarterly Journal of Economics, Nov. 1935, The Theory of Investment Once More: Mr. Boulding and the Austrians).

[3] Hayek, *The Mythology of Capital,* Quarterly Journal of Economics, February 1936. "I do not of course pretend that a fall of the rate of interest to zero is an event in the least likely to occur at any future

is difficult to see why diminishing returns should arise even in this very unreal sense, and it is impossible to find a basis for the theory that population growth can cause a retrograde development. The hypothesis of retrograde development seems intelligible only if we suppose that no rational anticipation of future wants is made: in these conditions a rise in the price of all existing capital goods would occur and a rise in the rate of interest would cause no increase in saving. The reproducible assets under these circumstances will be consumed or fall into disrepair, the non-reproducible will be valued on the basis of security or social prestige, apart from their income yielding capacity. But under these circumstances it would be impossible to consider the effect of any credit policy.

Is the rate of population growth therefore to be regarded as entirely irrelevant to the rate of investment? On the basis of any sort of equilibrium theory, excluding the period of production approach, it is impossible to treat it as influencing the rate of investment in a single direction. But though it may be denied that there is any a priori reason why an increasing rate of growth should cause a declining rate of investment, there is no reason to exclude the possibility of regarding the population increase as a sudden and unexpected change in the

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time in which we are at all interested. But, like all questions of what is *probable*, this is altogether irrelevant for theoretical analysis. What is of importance are the conditions under which this would be possible. Now if a condition were reached in which no further lengthening of the investment periods of individual ressources (either by lenghthening the process or by increasing the durability of goods in which they are invested) would lead to a further increase of output, new savings could not help to increase output. In the usual terminology, the marginal productivity of capital would have fallen to zero because no more satisfaction would depend on a particular capital good ("stored up labour") than would depend on the quantity of labour and other products which are needed to replace it. So long as any of the factors required for this purpose remain scarce, the capital goods themselves and *a fortiori* the final consumers' goods made with their help will remain scarce. And there can be no doubt that this point where further accumulation of capital would no longer increase the quantity of output obtainable from the factors used in its production, even if almost infinitely distant, would still be reached long before the point where no satisfaction whatever would be dependent on the existence of these factors".

data, which can cause a rise in the rate of interest. If previously the rate of saving has been sufficient to keep output per head stable, an unexpected increase in population growth will raise the rate of interest and — assuming continued uncertainty about the future — will change the preference of investors for certain types of assets on the basis of liquidity. In these circumstances more savings will be invested in the purchase of land and investment in other types of assets will be reduced, with the result that the high rate of interest will not call forth a sufficient increase in saving to reduce the rate.

To interpret the situation this way gives a much more reasonable interpretation of the effects of population growth. The root of the trouble in Poland is not the natural rate of growth, which is rapidly declining, but the decline in emigration. The rate of increase during the last ten years has been more rapid than it was in the period before the war because the number of emigrants per annum has declined to one tenth of the pre-war number, and this affects the situation, not merely as an increase in number of producers, but as a decrease in savings from emigrants, remittances (which in some districts in Poland used to form a large part of the total supply of savings).

If the State, under these circumstances, provides credits for the purchase of land and at the same time controls the purchase price of the land acquired by these means, it may conceivably cause an increased investment in other types of capital assets and an increase in the amount of savings [1]. This effect is problematical, if the liquidity preference is very strong. But whether it has this effect or not, such a policy certainly does not cause decapitalisation: the estate owner gets a lower price than the land is worth, but what it is worth does not depend on its future income yielding capacity.

Of course such a general argument as this is not intended to justify the administrative details of the reform. The methods by which it has been carried out are only important because they have limited its scope: the reform refers only to a proportion of the area under big farms, proceeds at a very slow rate, gives favourable terms to landowners and in its early

[1] cf. Keynes, *General Theory of Employment*, Interest and Money, p. 241.

stages the credits were given at too high rates of interest. (In fact about half the total transactions were financed by the savings of peasants themselves). Even a much more radical policy would not cause de-capitalisation. Complete expropriation, without compensation, would be another matter, since it would increase the uncertainty of investment, (and in this case the effect of sub-division of units could not be treated as negligible).

One view of the reform, which by reason of its complacency and bad economics, seems entitled to be described as the official view (as distinct from the political), is that a more radical policy is not worth attempting because the most complete sub-division possible would not "cure" over-population, i. e. would not give sufficient land to every peasant in Poland to provide him with full employment; in any case, why take any action, since it is only a matter of a generation or so before the new areas divided up will be overrun by increased population on the same low standard of living as before? Clearly this is the misconception which arises from Oberländer's error of regarding the earning capacity of an area of land as a fixed quantity and neglecting the possibility of increased investment. Though it is, of course, true enough that no sort of institutional change will cause any very considerable rise in incomes, it is obvious that any measure which might stimulate saving will contribute towards raising the standard in the future. What does seem to be an error of policy is to proceed with the reform as an one-sided investment, neglecting investment in improved communications, the absence of which forces all agricultural producers to keep up a higher degree of self sufficiency than is necessary.

Streszczenie

Doreen Warriner: Reforma rolna i problem kapitalizacji [1]

Rozpowszechniona jest opinja, że reforma rolna oznacza dekapitalizację t. zn. niszczy istniejące wartości kapitałowe

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i równocześnie stwarza dodatkowy popyt na kapitał ze strony nowych gospodarstw. Autorka, rozważając ten problem, definjuje dekapitalizację zgodnie z Hayekiem jako stopniowe niszczenie kapitału realnego przez nieodnawianie go lub używanie kapitału w sposób nierentowny. W obu przypadkach zmniejsza się przyszły dochód. Podział wielkich majątków zmniejsza wydajność kapitału inwestowanego w nich, niektóre rodzaje kapitału mogą stać się zupełnie bezużyteczne. Z drugiej strony podnosi się, że gospodarstwo drobne używa więcej pracy i kapitału na jednostkę powierzchni. Rozpatrując pierwszy z tych poglądów, należy rozpatrzeć zagadnienie optymalnej wielkości gospodarstwa. Istnienie obok siebie gospodarstw wielkich i małych dowodziłoby tego, że rozmiar przedsiębiorstwa rolnego nie jest ekonomicznie określony. Reforma rolna byłaby więc z ekonomicznego punktu widzenia obojętną. Optimum rozmiarów przedsiębiorstwa oznacza korzystne ustosunkowanie tych czynników produkcji, które są bardziej zmienne lub bardziej podzielne do względnie stałych lub niepodzielnych. Jako stałe elementy kosztów przyjmuje autorka dla gospodarstw małych koszt inwentarza żywego, zaś dla wielkich koszt budynków. I jeden i drugi typ gospodarowania osiąga minimum kosztów przy pewnym obszarze, przyczem któryś z nich musi być ekonomiczniejszy. Trwałe współistnienie ich obok siebie wynika z pewnych instytucjonalnych czynników, m. i. z tego, że włościanin dostarcza pracy i kapitału dla własnego gospodarstwa taniej niż na rynek. Jeżeli nawet przyjmiemy, że chłop żąda mniejszej rentowności swego kapitału inwestowanego w sile pociągowej niż przeciętna rynkowa rentowność, to i tak okaże się, że gospodarstwa drobne są przeinwestowane w tym kierunku.

O ile wielkie gospodarstwa są optymalne w stosunku do swych budynków, a drobne są zbyt małe w stosunku do swej siły pociągowej, przechodzenie ziemi z wielkiej do małej własności podwyższa koszta wielkich gospodarstw, a obniża u drob-

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analizy podstawowych zagadnień ekonomicznych, wiążących się ze sprawą agrarną.

Redakcja Studjów Ekonomicznych ogłasza je jako artykuł dyskusyjny w nadziei, że wywoła ono polemikę, która rzuci światło na jeden z najważniejszych problemów gospodarczych środkowej i wschodniej Europy.

nych, problem polega więc na tem, czy reforma rolna nie spowodowala zniszczenia kapitalu większej własności, większego niż wzrost wydajność małych. Fakty zdają się stwierdzać, że nie było ogólnej zniżki produktywności, wynikającej z parcelacji.

Najważniejszym skutkiem reformy rolnej nie jest jednak zmiana typu skali gospodarowania, lecz zmiana proporcji czynników produkcji. Reforma rolna zachęca do inwestowania w gospodarstwach małych, które używają więcej kapitału i pracy w stosunku do ziemi i kieruje więcej kapitalu do rolnictwa wogóle, t. zn. do tej gałęzi produkcji, która używa mniej kapitalu w stosunku do pracy niż przemysł. Polityka, zmierzająca do używania więcej pracy na jednostkę powierzchni, byłaby nieodpowiednia w Zachodniej Europie, gdzie postęp gospodarczy powoduje stały zwyżkowy kierunek płac i wynikające stąd oszczędzanie na pracy (wyludnienie wsi na Zachodzie). W Europie wschodniej wydaje się, że produkt na głowę ludności jest spadający lub najwyżej stały, ponieważ kapitalizacja nie wyprzedza przyrostu ludności. Rozdrabnianie wielkich gospodarstw było formą inwestowania oszczędności, które właściciel drobnego gospodarstwa lokuje w ten sposób z mniejszą zyskownością niż rynkowa. Proces ten jest nieunikniony o ile przyrost nowych oszczędności nie wystarcza na uprzemyslowienie. Reforma rolna przyspiesza go tylko, nie wpływa więc wogóle na zasób kapitału. Zmniejszony produkt na głowę wynika ze zbyt małej kapitalizacji, spowodowanej zdaniem niektórych autorów jak Oberländer i Poniatowski szybkim przyrostem ludności. Oberländer posługuje się tu pojęciem *Nahrungsspielraum* t. j. stałej zdolności produkcyjnej danego terytorjum, tak że każde zwiększenie liczby ludności powyżej pewnej granicy musi prowadzić do spadku dochodu na głowę. Poniatowski przyjmuje, że przeludnienie zachodzi, gdy przyrost ludności hamuje tak dalece kapitalizację, że dochód na głowę spada. Trudno jednak dojrzeć związek przyczynowy między przyrostem ludności a spadkiem stopy akumulacji. Pomiędzy akumulacją nowego kapitału, a momentem, w którym zaczyna on dawać zyski, istnieje przedział czasowy, w czasie którego ludność ulega zmianom, nie wyklucza to jednak możliwości przewidywania ogólnego kierunku zmian. Przewidywane przyszłe dochody z kapitału w porównaniu do kosztów, ponoszonych

w chwili oszczędzania, są podstawą kalkulacyjną oszczędzania. Jedynie przyjęcie ogólnego prawa zmniejszającego się przychodu z kapitału mogłoby uzasadnić tezę, że oszczędzanie jest stopniowo coraz trudniejsze. Dzisiejsza teorja kapitału jest dość niejednolita w tym punkcie. Teorja Knighta neguje prawo zmniejszających się przychodów od kapitału, teorja Hayeka dopuszcza to prawo jako teoretyczną możliwość. Autorka sądzi, że jedynie w przypadku, gdy jednostki nie antycypują racjonalnie przyszłych potrzeb, możliwy jest rozwój wsteczny i stały spadek dochodu na głowę. W takich warunkach dobra kapitałowe byłyby niszczone przez nieodnawianie, zaś ziemia jako niepotrzebująca amortyzacji byłaby ceniona na zasadzie bezpieczeństwa lokaty i pozycji społecznej, którą zapewnia jej posiadanie, nie zaś na podstawie zyskowności. Możnaby także uważać przyrost ludności za nagłą zmianę dat; w takim przypadku, o ile prowadzi do niepewności co do przyszłości, mógłby spowodować wzrost ceny ziemi i spadek inwestycji w innych działach produkcji. Przyczyną trudności w Polsce nie jest jednak stopa przyrostu ludności, która szybko spada, lecz zahamowanie emigracji, co wpływa nie tylko na ilość ludności ale i na podaż oszczędności, gdyż oszczędności emigrantów tworzyły przed wojną dużą część całej podaży oszczędności.

O ile państwo w tych okolicznościach kontroluje cenę ziemi i dostarcza środków na jej zakup, to wpływa to raczej na wzrost inwestycji w innych działach i na wzrost oszczędności. Jedynie gdyby dążenie do płynności było bardzo silne, mogłoby to nie spowodować wzrostu inwestycji. Polityka taka nie powoduje w żadnym razie dekapitalizacji.

Nawet znacznie bardziej radykalna reforma nie spowodowałaby dekapitalizacji, jedynie wywłaszczenie bez odszkodowania nie byłoby już obojętne, gdyż zwiększyłoby niepewność lokat. Wysuwa się zazwyczaj tezę, że najbardziej radykalna polityka nie może usunąć przeludnienia, gdyż nawet całkowity podział wielkiej własności nie mógłby dać wszystkim chłopom w Polsce pełnego zatrudnienia, a przy przyroście ludności następne pokolenie miałoby już poprzedni niski standart życiowy. Teza tu wynika z błędnego pojmowania (podobnie jak u Oberländera) produktywności ziemi jako stałej wielkości, zapomina

się tu, że istnieją zawsze możliwości zwiększenia ilości kapitału. Żadna zmiana instytucjonalna nie może oczywiście wydatnie zwiększyć dochodów, o ile jednak zachęci do oszczędzania, przyczyni się do wzrostu dochodu w przyszłości. Oczywiście konieczne są obok reformy rolnej także inne inwestycje, a zwlaszcza w komunikacji, trudności komunikacyjne zmuszają bowiem rolników do zwiększania samowystarczalności ponad rzeczywistą potrzebę.

JANUSZ LIBICKI

SOME CONSEQUENCES OF DIFFERENT INTERPRETATIONS OF SUPPLY CURVES

My attention was drawn to the problem which I propose to discuss in this paper by a footnote relating to Marshall's concept of particular expenses curve in Viner's "Cost curves and supply curves" (page 44, footnote 1). [1]

I have discussed this problem partly in my book on the theory of costs of production [2] in a chapter entitled "Cost curves and supply curves". I intend to discuss it here again in a somewhat different way.

The starting point of the following remarks is the Barone's interpretation of supply curve as given in his "Grundzüge der Theoretischen Nationalökonomie" [3] which is identical with Marshall's concept of particular expenses curve. On page 19, § 6 Barone says: "Die Beobachtung lehrt, dass auf einem Markt zu gleicher Zeit Unternehmer existieren, die das gleiche Produkt zu verschiedenen Kosten herstellen. Wenn wir diese Unternehmer nun in der Reihenfolge ihrer Kosten so anordnen, dass wir mit den niedrigsten Kosten anfangen und mit den höchsten aufhören, so können wir dies wie in Fig. 4 graphisch darstellen. Die Mengen OA, AB, BC sind die von den Unternehmern 3, 2, 1 hergestellten Warenmengen. Die entsprechenden Stückkosten werden von der Höhe der unschraf-

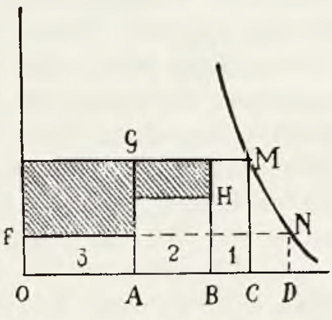
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[1] Zeitschrift für Nationalökonomie. Band III, Heft 1, 1931.

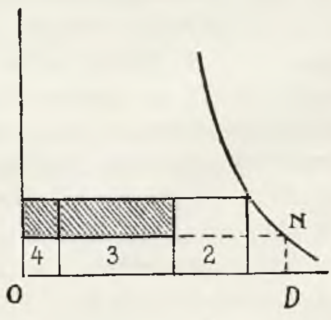
[2] J. Libicki, *Zarys teorji kosztów produkcji. (An outline of a theory of costs of production)*. Kraków 1935.

[3] Enrico Barone, *Grundzüge der theoretischen Nationalökonomie.* Übersetzt von Hans Staehle, Berlin und Bonn 1935.

fierten Rechtecke über *OA*, *AB*, *BC* bezeichnet. Weil nun die Produktion der Unternehmer 2 und 3 nicht ausreicht, um den Markt zu versorgen, besteht neben ihnen der Unternehmer 1 mit höheren Kosten. Die Unternehmer 2 und 3 machen Gewinne, die durch die schraffierten Rechtecke *GH* und *FG* dargestellt sind. Der Gleichgewichtspreis ist *MC*. Es trete nun ein anderer Unternehmer 4 am Markte auf, der zu gleichen Kosten wie der Unternehmer 3 produziert; sein Erscheinen wird unter Umständen die Unternehmung 1 vom Markte vertreiben. Es wird daraus eine Lage entstehen, wie Fig. 5 sie darstellt, in der die Unternehmung 2 an die "Grenze" getrieben ist. Als Ergebnis ist der Preis gesunken, die hergestellte und verbrauchte Menge gewachsen, der Gewinn des Unternehmers 2 zunichte geworden und derjenige des Unternehmers 3 vermindert. Kommt nun noch ein weiterer Unternehmer 5 hinzu, der zu den gleichen Kosten wie 3 und 4 produziert und den Unternehmer 2 vom Markte vertreibt, dann sinkt der Preis weiter; die verbrauchte Menge steigt und der Gewinn von 3, 4 und 5 neigt dazu zu verschwinden, indem der Preis sich den Kosten gleichstellt. Das ganze tendiert so zu dem Gleichgewicht, das in Fig. 5 durch die verbrauchte Menge *OD* und den entsprechenden Preis *ND* bezeichnet ist".



Barone, Grundzüge, Fig. 4



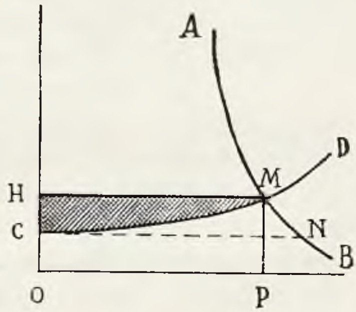
Barone, Grundzüge, Fig. 5

"Diese Betrachtung erklärt den scheinbaren Widerspruch zwischen der realen Existenz des Unternehmergewinns (der theoretisch nur als vorübergehende Erscheinung betrachtet werden kann) und der Tendenz des freien Wettbewerbs, den Preis auf die Produktionskosten zu reduzieren, damit also den Unter-

nehmergewinn zunichte zu machen. Natürlich ist den Produktionskosten in diesem Sinne der Unternehmerlohn einzurechnen".

"Das Stufen-Diagramm der Fig. 4 und 5 kann, wenn es sich um einen grossen Markt und um eine grosse Anzahl konkurrierender Unternehmer handelt, in eine kontinuierliche Kurve *CD* (Fig. 6) übergehen. Wir werden diese Kurve *CD* immer die Kurve des Gesamtangebots (offertà complessiva) nennen, wie andererseits *AB* die Kurve der Gesamtnachfrage darstellt".

"Es leuchtet ohne weiteres ein, dass erstens die Fläche *MHC* (Fig. 6) die Summe der Unternehmer-Gewinne darstellt, und dass zweitens die Konkurrenz der Unternehmer dahin tendiert, den Punkt *M* mit dem Punkt *N*, die Kurve *CD* also mit der Geraden *CN* zusammenfallen zu lassen und damit die Fläche *MHC* zu null zu reduzieren".



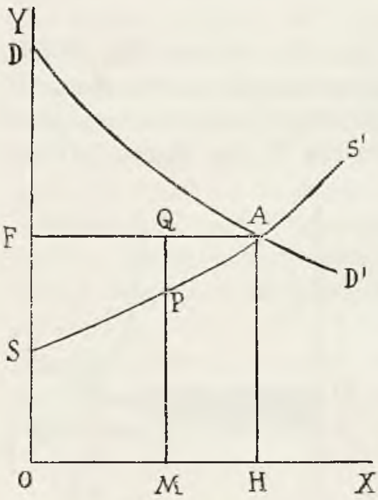
Barone, Grundzüge, Fig. 6

"Um die Theorie des Angebots zu verstehen, muss man sie sich in der eben aufgeführten Weise vorstellen, dass heisst die Unternehmer in der Reihenfolge ihrer Selbstkosten anordnen".

I quoted Barone here at length to show clearly that what he means by his supply curve (die Kurve des Gesamtangebots) is identical with what Marshall calls "particular expenses curve". To use Marshall's own words:

"In the adjoining diagram, *SS'* is not a true supply curve adapted to the conditions of the world in which we live; but it has properties, which are often erroneosly attributed to such a curve. We will call it the particular expenses curve. As usual the amount of a commodity is measured along *Ox*, and its price along *Oy*. *OH* is the amount of the commodity produced annually *AH* is the equilibrium price of a unit of it. The producer of the *OHth* unit is supposed to have no differential advantages; but the producer of the *OMth* unit has differential advantages which enable him to produce with an outlay *PM*, a unit which it would have cost him an outlay *AH* to produce without those

advantages. The locus of P is our particular expenses curve; and it is such that any point P being taken on it, and PM being drawn perpendicular to Ox, PM represents the particular expenses of production incurred for the production of the OMth unit. The excess of AH over $PM = OP$, and is a producer's surplus or rent. For convenience the owners of differential advantages may be arranged in descending order from left to right; and thus SS' becomes a curve sloping upwards to the right.



Marshall, Principles. Fig. 25

Proceeding as in the case of consumer's surplus or rent, we may regard MQ as a thin parallelogram or as a thick straight line. And as M takes consecutive positions along OH, we get a number of thick straight lines cut in two by the curve SA, the lower part of each representing the expenses of production of a unit of the commodity, and the upper the contribution which that unit affords towards rent. The lower set of thick lines taken together fill up the whole space $SOHA$; which therefore represents the aggregate of the expenses of production of an amount OH. The upper set of thick lines taken together fill up the space FSA, which therefore represents producer's surplus or rent in the ordinary sense of the term. Subject to the corrections mentioned above DFA represents the surplus satisfaction which consumers get from an amount OH over that, the value of which is represented to them by a sum of money equal to $OH \cdot HA$; and the diagram shows how the name "consumer's rent" was suggested for this surplus.

Now the difference between the particular expenses curve and a normal supply curve lies in this, that in former we do, and in the latter we do not, take the general economies of production as fixed and uniform throughout. The particular expenses curve is based throughout on the assumption that the

aggregate production is *OH*, and that all the producers have access to the internal and external economies which belong to this scale of production; and, these assumptions being carefully borne in mind, the curve may be used to represent a particular phase of any industry, whether agricultural or manufacturing: but they cannot be taken to represent its general conditions of production.

That can be done only by the normal supply curve, in which *PM* represents the normal expenses of production of the *OMth* unit on the supposition that *OM* units (not any other amount, as *OH*) are being produced; and that the available economies of production external and internal are those which belong to a representative firm where the aggregate volume of production is *OM*. These economies will generally be less than if the aggregate volume of production were the larger quantity *OH*; and therefore, *M* being to the left of *H*, the ordinate at *M* for the supply curve will be greater than for a particular expenses curve drawn for an aggregate production *OH*.

It follows that the area *SAF* which represents aggregate rent in our present diagram would have represented something less than aggregate rent, if SS' had been a normal supply curve even for agricultural produce (DD' being the normal demand curve). For even in agriculture the general economies of production increase with an increase in the aggregate scale of production.

If however we choose to ignore this fact for the sake of any particular argument; that is, if we choose to assume that *MP* being the expenses of production of that part of the produce which was raised under the most difficult circumstances (so as to pay no rent) when *OM* units were produced, it remains also expenses of production (other than rent) of the *OMth* unit even when *OH* is produced; or in other words, if we assume that the increase in production from the amount *OM* to the amount *OH* did not alter the expenses of production of the *OMth* unit, then we may regard *SAF* as representing the aggregate rent even when *SS* is the normal supply curve. It may be occasionally convenient to do this, attention being of course called every time to the nature of the special assumption made" [1].

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[1] A. Marshall, *Principles of Economics*, London 1898, p. 521.

As can be seen from the above quotation, though Marshall, distinguishes clearly between the supply curve and the particular expenses curve yet his distinction is sometime more of a quantitative than of a qualitative character and he does not take notice of all the difference between these two concepts.

As Viner says in the footnote mentioned above: "It will be noticed that his particular expenses curve, SS', is drawn so as to project somewhat beyond the point of total output for the industry as a whole A. This is an error, and no signiffieance can be given to the part of the curve projecting beyond the point of total output of the industry as a whole. If the output of the industry were to increase up to the terminal point of this curve, the entire curve would acquire a different locus".

My first task is to give a quite precise meaning to Viner's rather general statement.

Let us denote by $q_1\ q_2\ q_3 \dots q_n$ the amounts of a given good produced by entrepreneurs 1, 2, 3, ... n, by $f_1(q_1)$, $f_2(q_2)$, $f_3(q_3)$, ... $f_n(q_n)$ their respective costs of production. The average unit cost for entrepreneur 1 will then be $\frac{f_1(q_1)}{q_1}$, for entrepreneur 2 $\frac{f_2(q_2)}{q_2}$, and so on. Their respective marginal costs will be $\frac{df_1(q_1)}{dq_1}$, $\frac{df_2(q_2)}{dq_2}$, ... $\frac{df_n(q_n)}{dq_n}$. P is the price of the good produced. The entrepreneurs will maximize their individual profits if they will act according to the formula $P = \frac{df(q)}{dq}$. By q^0 I will denote the individual output at which the average unit cost is minimum.

Let us assume now that entrepreneur 1 has the lowest unit costs and entrepreneur n the highest. Entrepreneur 1 will start his production only when price will at least equal his minimum average unit cost. If this will be case, his production will equal q_1^0, his price will equal the minimum average unit cost and the marginal cost:

$$P = \frac{f_1(q_1^0)}{q_1^0} = \frac{df_1(q_1^0)}{dq_1^0}$$

Let us assume now that the price goes higher up so that entrepreneur 2 starts his production. (Entrpreneur 2 will enter the field of production only when the price will be high enough to cover at least his minumum average unit cost). The formula: price = minimum average unit cost = marginal cost, will now be true for entrepreneur 2 but will not hold true any longer for entrepreneur 1. For entrepreneur 1 the price will be now higher than both his average and his marginal cost. His profit (or producer's rent) will equal the difference between his gross receipts and his total costs:

$$R = Pq_1^0 - f_1\ (q_1^0)$$

His marginal cost being lower than the unit price of the good produced, the situation of entrepreneur 1 will not comply now with the formula

$$P = \frac{df\ (q)}{dq}$$

and in order to maximize his profit R he will have to increase his production to such an amount (q_1') at which his marginal cost will equal the new and higher price. The total volumen of production will now be larger not only by the amount produced by entrepreneur 2 but also by the difference between the previous and the present production of entrepreneur 1.

Entrepreneur 1 will now produce more than before, his marginal cost will now be higher than before and his present average unit cost will be higher than previously. His profit will be:

$$R = Pq_1' - f_1\,(q_1')$$

Entrepreneur 2 will produce the amount q_2^0 at his lowest average unit cost equal to the price and will realize no extra profit. He will assume now the rôle of the "marginal entrepreneur". A quite analogical development must always take place with every change of price and with every increase (or decrease) in the number of entrepreneurs.

If we assume now that we have not two but n entrepreneurs, every change of the price will result not only in the change of the total volumen of production but will affect the amounts

produced individually by every entrepreneur and consequently their marginal and average unit costs. Every rise in the price of the good produced must result in the expansion of the production of all and each individual entrepreneur and will tend to raise their marginal and average costs. Every fall in the price of the good produced must result in the contraction of the production of all and each individual entrepreneur and will tend to decrease their marginal and average costs.

It follows from the above that the Baronian supply curve as well as the Marshallian particular expenses curve does not represent the presumed reactions of the entrepreneurs on the price changes. Both curves depict only the way in which the entrepreneurs have reacted on the last price-change and how they have adjusted their production policies to the actual price. Both curves tell us nothing of how the entrepreneurs will react on a future (higher or lower) price. This is the reason why the Baronian supply curve or the Marshallian particular expenses curve can not be drawn so "as project beyond the point of total output of the industry as a whole" and why if the output of the industry as a whole were to change "the entire curve would acquire a different locus".

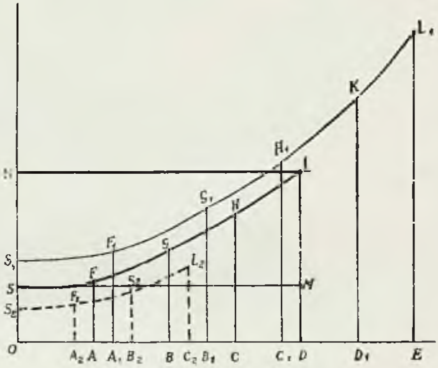
Graph 1 represents the above. *OA*, *AB*, *BC*, *CD*, are the amounts produced by entrepreneurs 1, 2, 3, 4, at the price *P* equal *DL*. Their respective average unit costs are. *AF*, *BG*, *CH*, and *DL* and their marginal costs are equal to the price *P* equal *DL*. The thick curve *SL* represents then the Baronian supply curve or the Marshallian particular expenses curve at the price *P* equal *DL*. This curve represents then the final adjustment of the production policies of all entrepreneurs to the given price of the good produced by them.

As it can be easily seen from graph 1 the entrepreneurs are arranged according to the level of their average unit cost of production. Entrepreneur 4 is the "marginal entrepreneur" and the surface enclosed between the curve *SL*, the y-axis, and the priceline *NL* represents the extra profits of entrpreneurs 1, 2, and 3.

If the price were to rise to the level P_1 equal EL_1 all the entrepreneurs would increase their individual outputs: entrepreneur 1 from *OA* to OA_1 entrepreneur 2 from *AB* to A_1B_1;

3 from BC to B_1C_1 and 4 from CD to C_1D_1 and entrepreneur 5 would start his production in the amount D_1E. Their respective average unit costs would now be: A_1F_1, B_1G_1, C_1H_1, D_1K, EL_1; and their marginal costs would equal the new price P_1 equal EL_1. The marginal entrepreneur would be now entrepreneur 5 who will realize no extra profits. The Baronian supply curve or the Marshallian particular expenses curve would be now represented by the thin curve S_1L_1.

If, on the other hand, the price were to decrease to the level P_2 equal C_2L_2 all the entrepreneurs would contract their individual outputs: entrepreneur 1 from OA to OA_2 entrepreneur 2 from AB to A_2B_2; 3 from BC to B_2C_2 and entrepreneur 4 would altogether stop his production. Their respective average unit costs would now be: A_2F_2, B_2G_2, C_2L_2; and their marginal costs would equal the new price P_2 equal C_2L_2. The marginal entrepreneur would be now entrepreneur 3 who will realize no extra profits. The Baronian supply curve or the Marshallian particular expenses curve would be represented in this case by the dotted curve S_2L_2.



Graph 1

If we analize further the construction of these curves we must come to the conclusion that they do not represent the state of final equilibrium. For there are two possibilities: either the differences in the level of costs of production among entrepreneurs are of a technical character or are rather of such a nature as for instance in agricultural production. In the first case when the differences are differences in the technique or organization of production, the competition tends to equalize the conditions of production and the costs of production on the level of the best organized and cheapest producing enterprise. When this process is closed there are no more differences in existence among the entrepreneurs, there is no "marginal entrepreneur",

nobody is realizing extra profits, and the formula: price = marginal cost = lowest average unit cost, holds true for all and each individual entrepreneur.

In the second case the equalization must also take place but in quite another way. When the differences are of such a nature that they can not be equalized by free competition among entrepreneurs the different producers rents will remain unchanged. In this case however the extra profits must be capitalized according to the current interest rate and added to the original value of the enterprise. The different values of particular enterprises will be established according to their rentability. The extra profits (or the producers rents) will become now the normal interest of the additional value of the more rentable enterprise. The increase or decrease in the extra profits will be immediatly compensated by the corresponding increase or decrease in the value of the enterprise.

In the second case therefore the costs of production of different enterprises will be equalized on the level of the most expensively producing enterprise or, in other words, on the level of the marginal entrepreneur. When this process is closed there are no more differences among the entrepreneurs, there is no marginal entrepreneur, nobody is realizing extra profits, and the formula: price = marginal cost = lowest average unit cost holds true again for all and each individual entrepreneur.

In both cases the curve SL tends to acquire the position of a straight line parallel to the x axis. In the first case the curve SL will finally become the straight line SM; in the second case the curve SS' will finally become the straight line NL. We can call therefore the curve SL the short-run and the straight line SM (or NL) the long-run Baronian supply curve.

With every increase of the price of the produced good the whole straight line SM (or NL according to the case) will move upward and with every decrease of the price it will mowe downward.

It must be noted that with every move upward the whole curve SL or the straight line SM (or NL) will lengthen and with every move downward will shorten so as to correspond always with a greater output at a higher price and a smaller output at a lower price.

Such must be the behaviour of a curve based on the concept of average unit costs and constructed in such a way as are constructed the Baronian supply curve and the Marshallian particular expenses curve. The main advantage of such a construction is that only that construction gives us a possibility to represent graphically the producer's rent. Its main disadvantage is that it represents only the momentary situation as resulting from the actual price and if the demand were to change, it tells us nothing about what would be the new equilibrium price and the new equilibrium output for the industry as a whole.

Such are the consequences of interpreting the supply curve as based on average unit costs. What consequences must we take into account if we are to interpret the supply curve as based on marginal costs?

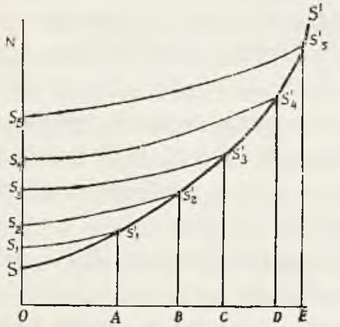
In the free competition system the price is a given datum and every entrepreneur must adjust his output so as to comply with the formula: price = marginal cost. As the formula must hold true for every entrepreneur it results that all the entrepreneurs are producing their parts of the total output at the same marginal cost. Every point on such a curve represents therefore the marginal cost, the same for all and each individual entrepreneur and corresponding to a given total output. With every change of price every entrepreneur adjusts his output so as to comply with the formula price = marginal cost. With every increase of the price, therefore, he expands his production and produces more at a higher marginal cost equal to the hihger price and with every decrease of the price he contracts it and produces less at a lower marginal cost equal to the lower price. A supply curve based on the marginal cost we can call also a supply-price curve. Every shift of the demand-price curve brings forward a corresponding change of the total output and of the supply-price.

The main advantage of such a construction of supply curve is that with every change of the demand curve we can easily find the new equilibrium output for the industry as a whole and a new equilibrium price. Its main disadvantage is however that it tells us nothing about the producer's rent. We have no right now to draw from the point of intersection of both curves (the equilibrium-price point) a line perpendi-

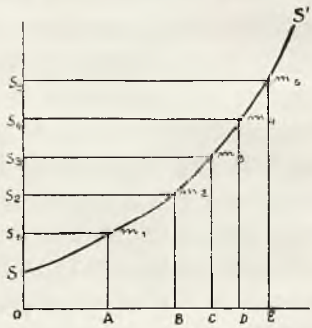
cular to the y-axis as we could do in the case of the Baronian supply curve or in the case of the Marshallian particular expenses curve.

We can not do it now because of the fact that the market supply-price curve is not a sum of the individual marginal cost curves of particular entrepreneurs. The integral of the market supply-price function does not equal, therefore, the sum of total costs of production of all the entrepreneurs. The surface enclosed between the curve, the y-axis, and the price-line which represented in graph 1 the producer's rent has no meaning at all in a graph representing a supply curve based on marginal costs.

The relation between a supply curve based on the average cost and a supply curve based on the marginal cost is represented on graphs 2 and 3.



Graph 2



Graph 3

The curves $s_1s'_1$, $s_2s'_2$, $s_3s'_3$, $s_4s'_4$, $s_5s'_5$, on graph 2 represent the so-called (page 23) short-run Baronian supply curves or the Marshallian particular expenses curves (based on the average unit cost). The straight lines s_1m_1, s_2m_2, s_3m_3, s_4m_4, s_5m_5, on graph 3 represent the so-called (page 23) long-run Baronian supply curves.

The terminal points of the curves $s_1s'_1$, $s_2s'_2$, and so on (or of the straight lines s_1m_1, s_2m_2, ...) represent the prices = the lowest average unit costs of marginal entrepreneurs = the marginal costs the same for all and each individual entrepreneur. The line joining these terminal points represents then the mar-

ginal costs at which each entrepreneur produces his part of the total output of the industry as a whole measured on the x-axis.

The line joining the terminal points of all the short- (or long-) run Baronian supply curves is therefore nothing else but the market supply-price curve based on the marginal cost.

Thus on both graphs the curves SS' represent the supply curves based on marginal cost. (The market supply price curves).

From graph 2 it can be easily seen why the supply curve based on marginal costs can not explain the producer's rent. Let us take for instance the situation represented by the short-run Baronian supply curve $s_5 s'_5$. The gross receipts of all entrepreneurs are represented in this case by the rectangle ONs'_5E the total costs of production of all entrepreneurs by the surface $Os_5s'_5E$ and the total producers' rent by the surface $s_5Ns'_5$. Thus only the surfaces limited by the x and y- axed and a curve based on the average costs have a meaning. The surfaces limited by the x and y- axes and a curve based on marginal costs, that is, the surface OSs'_5E or the surface SNs'_5 have no meaning at all.

To sum up shortly what was said above: a supply curve based on the average unit cost helps us to understand the producer's rent and its changes brought forward by the changes of the price of the produced good, but it can not explain to us the shift from one equilibrium price to another equilibrium price caused by the shift of demand; on the other hand a supply curve based on the marginal cost enables us to determine every equilibrium price and every equilibrium output for the industry as a whole as resulting from every shift in demand; it can not however explain to us the producer's rent.

Streszczenie

Janusz Libicki: Pewne konsekwencje różnych interpretacyj krzywych podaży

Punktem wyjścia poniższych rozważań jest uwaga Viner'a w jego artykule p. t. "Cost curves and supply curves" (Zeitschrift für Nationalökonomie); odnosząca się do Marshall'owskiego pojęcia krzywej kosztów specjalnych. Z zestawienia cytat z Marshall'a "Principles of Economics" oraz Barone'go "Grund-

züge der theoretischen Nationalökonomie" wynika, że krzywa podaży w interpretacji tego ostatniego pokrywa się zupełnie z pojęciem krzywej kosztów specjalnych Marshall'a. Jakie są plusy i minusy tego rodzaju ujmowania krzywej podaży? Jeśli uszeregujemy wszystkich przedsiębiorców według wysokości ich jednostkowych kosztów produkcji, otrzymamy w rezultacie krzywą rosnącą od lewej ku prawej. Mając daną wielkość ogólnej produkcji, możemy wówczas oznaczyć t. zw. krańcowego przedsiębiorcę oraz przeprowadzając przez punkt, odpowiadający danej cenie, prostą równoległą do osi X-ów, możemy oznaczyć wielkość sumy rent producentów. Znając wielkość produkcji danego przedsiębiorcy oraz jego miejsce w szeregu, możemy na tak skonstruowanym diagramie oznaczyć wielkość jego indywidualnej renty producenta. To jest największy plus tego rodzaju konstrukcji. Jej minusem jest natomiast to, że tak skonstruowana krzywa reprezentuje tylko pewną sytuację ściśle określoną daną ceną. Z chwilą natomiast, gdy krzywa popytu ulegnie przesunięciu (w górę lub w dół), nie jesteśmy już w stanie określić ani nowej ceny równowagi, ani nowej wielkości produkcji. Przy każdej zwyżce ceny bowiem cała tak skonstruowana krzywa podaży zostaje przesunięta w górę i przedłużona, oraz przy każdej obniżce ceny zostaje przesunięta w dół i skrócona. Zwrócić należy jeszcze uwagę na to, że różnice w wysokości kosztów jednostkowych u poszczególnych przedsiębiorców mogą wynikać albo z różnic w poziomie techniki produkcyjnej, albo z różnic, wynikających z ograniczoności pewnych czynników produkcji, jak np. w rolnictwie. W pierwszym wypadku wolna konkurencja doprowadza do rozpowszechnienia danej metody i organizacji produkcyjnej i tem samem dąży do wyrównania jednostkowych kosztów produkcji na poziomie najtaniej produkującego przedsiębiorcy. W rezultacie więc tak skonstruowana krzywa podaży stać się musi w stanie równowagi linją prostą, równoległą do osi X-ów, a renty poszczególnych przedsiębiorców muszą zaniknąć. W wypadku drugim zaś, renty poszczególnych przedsiębiorców muszą zostać skapitalizowane według bieżącej stopy procentowej i dodane do wartości poszczególnych przedsiębiorstw. Z rent stają się więc zwykłem oprocentowaniem zwiększonej wartości przedsiębiorstwa. I w tym więc wypadku krzywa podaży przemienia się w linję prostą.

Z innemi konsekwencjami mamy do czynienia wówczas, gdy konstrukcję krzywej podaży opieramy na pojęciu kosztu krańcowego. Ponieważ wszyscy przedsiębiorcy produkują po tym samym koszcie krańcowym (nawet wówczas, gdy produkują po różnym koszcie jednostkowym) równym cenie, każdy punkt na krzywej podaży określać będzie przy jakiej cenie, jaka ilość i po jakim koszcie krańcowym będzie dostarczane na rynek. Z tak skonstruowanej krzywej podaży zawsze możemy odnaleźć nową cenę równowagi i odpowiadającą jej wielkość produkcji. Tak skonstruowana krzywa podaży nic nam jednak nie mówi o rencie producenta i o jej zmianach. Nie nadaje się więc do wytłómaczenia tego pojęcia.

JAN DREWNOWSKI

IMPERFECT COMPETITION AND THE CONSUMER[1]

1.

A great proportion of the new work done recently in economic theory is essentially a reconsideration of old problems on more realistic assumptions. Quite a number of subjects have been dealt with and put into new form. But no doubt there still remain many problems to be worked out along these new lines.

The purpose of the present paper is to deal with one of these problems, namely with the consumer's situation under realistic assumptions, frequently referred to as imperfect market conditions.

Prima facie the conditions in which a consumer finds himself under a system of imperfect competition[2] may not seem to be very different from those of the most "perfect" Paretian system; but in fact the differences are quite important.

Among the assumptions that have been dropped in order to pass from one system to the other, there is first that of an infinite number of competing entrepreneurs. The second assumption we drop, when passing from a perfect to an imperfect market, is the assumption of the uniformity of the product.

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[1] This article is in the main a development of chapter VI of a thesis by the present author *O niedoskonałym popycie (On Imperfect Demand)*, which is to be published shortly in Polish.

[2] The term "imperfect competition" is used here and will be used troughout this paper in the most usual sense, which corresponds rather to what Prof. Chamberlin calls "monopolistic competition" than to this imperfect competition proper. The difference between these concepts seems to me to be in degree rather than in kind.

The differentiation of the product is usually defined as a discrimination by the buyer between products of different sellers on some real or fancied basis [1].

Having dropped these two assumptions we have reached the imperfect competition level of abstraction as it is usually defined. But there are some further characteristics of the systems of perfect and imperfect competition which should be taken into account. These are the different degrees of certainty which are assumed in connection with these systems.

In a system of perfect competition, as it is defined in the "Appendice" to Pareto's "Manuel", we have a perfect subjective certainty and no changes anticipated. In other words in such a system we assume the individual being perfectly certain that the data will not change [2].

In imperfect conditions of the market we usually assume some anticipation on the part of the individual [3], but two alternatives are possible as far as certainty is concerned.

We may either assume the individual anticipating changes with perfect certainty or drop the assumption of certainty and conceive the individual as anticipating changes, but with uncertainty [4].

2.

Market Imperfections. Product Differentiation

Let us examine now what situation arises after we have dropped the two assumptions distinguishing perfect from imperfect competition, i. e. the infinite number of entrepreneurs and the uniformity of the product. For the moment let us leave uncertainties aside.

Dropping the assumption of the infinite number of entrepreneurs has, as we know, quite a great importance for every individual entrepreneur, because it enables him to have some

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[1] E. Chamberlin, *The Theory of Monopolistic Competition*, p. 56.

[2] This does not imply that he is necessarily right. If he were so, this would mean that he has a perfect knowledge of the future, an assumption which we are never entitled to make. See par. 4 below for a further development of this point.

[3] This is not always stated explicitly, but nearly always implied.

[4] See par. 4 below.

influence on the market price, and consequently leads to a different equilibrium than would be reached in perfect competition.

From the point of view of the consumer, however, this change does not seem to be important. It will result usually in a change in prices, which would not be different for the consumer from a change of data resulting from any other reason.

The introduction of the differentiation of the product will have some effects for the consumer as well as for the entrepreneur [1].

The appearance of discrimination on the part of the buyers gives every entrepreneur some special market of his own and a possibility of differentiating his product in some other way by changing its physical properties. He may, so to speak, adjust his product to the demand of the public. Consequently he will himself differentiate his products in the same way as the buyers do. What they call different products will be different products to him, no matter what are the physical properties of these products.

Let us turn now to the individual consumer.

There are consequences of buyers' differentiations that are relevant entrepreneurs and affect consumers but indirectly; these are mentiomed above.

There is however one consequence of this differentiation which affects the consumer directly: this is the fact that second hand goods fetch as a rule a lower price than new goods. From this it follows that the consumer may buy some good at one price, but may sell the same good at a lower price only [2].

Let us examine this situation more closely. Every consumer has in his possession some property. We express this by

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[1] For a more detailed examination of this problem see *The Classification of Commodities and the Problems of Competition and Monopoly* by the present author in the II issue of the Studia Ekonomiczne.

[2] This seems to be a commonplace, but none the less its implications are seldom fully realised. The only detailed analysis of this problem which I was able to find is in Carl Menger's *Grundsätze der Volkswirtschaftslehre*, Wien 1923, chapter VIII, pp. 217—240, and also in his article *On the Origin of Money* in "The Economic Journal" 1892, pp. 243 sq. He looks however at this problem from a different point of view and consequently his concept of saleableness, valuable as it is, could not be of great use for the questions discussed in this paper.

an indifference system which has many dimentions along which are measured quantities of separate goods. The question arises how the property of the individual is to be devided into separate goods. There may be conceived perhaps a few ways of doing it, but the simplest one which is implied in most of the discussions of this question, is the division according to the individual's own classification. Because undoubtedly the individual has in his mind some sort of classification of his property into separate goods. Consequently the representation of the quantities of the goods possessed along the co-ordinates of the preference system corresponds closely to reality.

If we now take a good in possession of the individual, unambiguously defined as one single good, and compare its definition with that which is accepted in the market, we shall see a definite divergence.

If the consumer wishes to buy some more of this good, he will buy in the market what is called, say, good Y, but if he wishes to sell a part of his good Y, he will sell what is called a second hand good Y, and considered a thing definitely distinct from the new good Y.

The important point that arises here is that there is no longer any uniformity in the differentiation of goods. What the consumer considers to be one single good is considered in the market to be two separate ones [1].

The same thing may be expressed by saying that the individual is confronted with two markets. In one of them he may buy, and in the other he may sell. But this would be a mere change of terminology, because the statement that the consumers differentiate utween products of different sellers comes to the same thing as the statement that different sellers have separate markets [2].

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[1] The opposite case is also conceivable. What is in the market one good may be considered as two goods by the consumer. This is the case of one good having two separate uses for the consumer. This is however a problem definitely distinct from that discussed in the present paper and not relevant in this connection. It will therefore not be discussed here.

[2] The phenomenon itself is of course a very well known one, as its essence lies in the fact that second hand goods as such have a lower price than the new ones. It might have been perhaps introduced without the

3.

Let us represent the state of things described in the previous paragraph by a consumer's indifference diagram (fig. 1). Along the X axis we shall measure money, which is never second hand, of course,[1] and some other good along the Y axis.

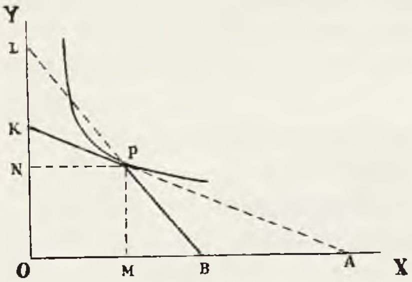


Fig. 1

If our individual is staying ot the point P, he is confronted with two prices. One of them is expressed by the gradient of the line KP, and applies to the case when he wishes to buy Y. If he wishes to sell Y he is compelled to move along the line PB the gradient of which represents the price of Y in the second hand goods market.

The two price lines KP and PB therefore form an obtuse angle with its vertex at P; this angle takes the place of the straight line which we used to represent the price under perfect competition.

This phenomenon may be called the refraction of prices. We may attempt to express it more precisely by introducing the concept of the rate of refraction. This will be the proportion of the market price to the "refracted" price or AM/MP: BM/MP (fig. 1).

The ordinary market price and the refracted price are in some way connected with each other. We may say that when they are changing it is usually in one direction. We may also contend that the rate of refraction increases when the price increases. But both these statements express only some tendencies which are not at all necessarily fulfilled. The market

preliminary remarks whinh took a great part of this paragraph. They were needed however to show that this phenomenon is implied in and may be deduced from the assumption of the differentiation of the product and must be recognised therefore in a reasoning on the imperfect competition level of abstraction.

[1] See par. 5 below for a further development of this point.

price and the refracted price are, as we know, prices in separate markets and consequently they may depend on forces that are entirely independent of each other. The rate of refraction may have every value from 1 to infinity.

4.

Uncertainties.

So far the question of uncertainty has not been taken into account in our discussion. Let us proceed now to this subject and examine the relations between refracted prices and uncertainties.

Under the term uncertainty we shall understand always what may be more precisely described as subjective uncertainty i. e. the individual's own estimate of uncertainty, as distinguished from risk which is objectively calculable [1].

Uncertainty may always be conceived as an uncertain possibility of changes of data relevant to the individual, on the condition that these data will include really all the data of sifnigicance to him [2].

In a system of perfect competition, as described by Pareto there is perfect certainty throughout and no changes of data anticipated. Given the tastes of the individuals and the resources we may easily find what distribution of the resources will take place, and therefore determine the position of stable equilibrium.

In such a system the existance of the refractions of prices would have no significance whatever. As nobody foresees any changes of data the conditions of re-selling the goods he is now acquiring will not affect his present purchases at all. We may

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[1] Cf. Knight, *Risk, Uncertainty and Profit*, p. 19 and ch. VII. Another form of expressing uncertainty may be the statement that when uncertainty is present the individual's economic judgment has the form of a mean value and some measure of dispersion instead of a single value. J. R. Hicks, *Suggestion for Simplifying the Theory of Money*, Economica 1935, p. 8.

[2] These possible changes of data may be of three rather distinct types. The first is a purely subjective one viz. it will concern changes in the individual's tastes. The second type will include changes in the individual's property or income not resulting from his acts of choice but coming from outside. The third type of change is the change of prices in the market.

say therefore that at this level of abstraction the refractions of prices do not exist.

The next, more realistic degree of approximation will be a system in which we shall have a perfect certainty, but in which there are changes of data anticipated [1].

If in such a system we are given all the actual data and their anticipations, we may find the position of equilibrium. In this case however the refractions of prices have a definite significance. As changes are anticipated, the possibilities of re-selling must be anticipated too, and therefore the refracted prices taken into account.

When we pass to a still more realistic system where are anticipated changes with uncertainty, the refractions of prices become still more important. Uncertainty is connected with every purchase that is made. Consequently the re-selling price should be taken into consideration in connection with every good that is acquired. Whatever change of data appears, it results in some adjustment taking place, and this adjustment will often have to be carried out at a refracted price.

From this it follows that the existence of the refractions greatly increases the importance of uncertainties to the individual.

We may go however even further than that, and say that in many cases the existence of refractions is a condition without which uncertainty would not be of significance to the individual at all.

There are changes of data which mean a loss to the individual. There are others which mean a gain to him. But there

[1] This system is really only slightly different from the previous one. The essential difference is only that it is a more general one, of which the former may be said to be a particular case. From our point of view there are however reasons for a distinction between them. Firstly because the first system only is used in the traditional theory of perfect competition (e. g. Pareto) and therefore when we pass from the first to the second system we abandon the assumptions of the traditional theory, and secondly because this transition proves to be particularly important for our special problem i. e. for the refraction of prices, which is not relevant in the first, but is so in the second system.

are also changes which bring neither loss nor gain to the individual but are a matter of indifference to him [1].

If refraction did not exist at all the individual would not care much about the possibilities of this sort of change and the uncertainties connected with it. Whether a change was anticipated at once, or whether a wrong anticipation is to be corrected afterwards by some adjustment, this would not matter to him, as all the adjustments would be performed without any loss on his part.

The situation changes diametrically when refracted prices are introduced. It is no longer a matter of indifference to the individual whether he foresees rightly or wrongly, because if he is wrong some adjustment would be necessary and would often be carried out at a refracted price that is at a loss to him. The estimates of uncertainties become therefore relevant to the individual as a result of refractions of prices.

We may say generally that the existence of refractions increases the importance of uncertainty to the individual, and sometimes even makes uncertainty relevant where it would not be so if there were no refractions. In any case a high rate of refraction makes every good more "uncertain" because of a greater loss in the event of re-selling it.

5.

The Store of Value Function of Goods.

The refraction of prices has some relevance to the so-called "store of value" function of goods. Let us examine this relation in more detail.

The "store of value" function becomes relevant when the

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[1] Changes of data to which the individual may be said to be indifferent are firstly all changes of the individual's tastes. In this case the old and new systems of preferences cannot be compared. The change itself must be therefore said to bring neither loss nor gain. What matters are the conditions of adjustment, and these in the case of refractions will be less favourable than if refractions were absent. The second and the third type of change (as classified above in the note page 10) will as a rule bring some gain or loss to the individual. There may however be some changes of the third type to which the individual is indifferent, but these will be rather special cases.

individual finds himself confronted with uncertainties. Its essence lies in the fact that the possession of some durable goods acts against uncertainty i. e. against the detrimental results of uncertain changes of data.

The store of value function would be performed ideally by a good with which no uncertainty is connected and having no refractions whatever. It is obvious however that such an ideal good cannot exist in practice.

As far as refractions are concerned there is one good which has the rate of refraction equal to unity, i. e. not subject to refraction at all. This is money which is a means of exchange, and the very essence of its function consists in not having any refraction i. e. in being accepted on equal terms by everybody from everybody [1].

Besides money we may find other goods having a very small rate of refraction. The rate equal to unity is often possessed by objects having artistic value, such as pictures etc. which are not less appreciated because second hand.

In general durable and not specialised goods would have a rate of refraction rather small compared with easily perishable and very specialised ones.

The highest rate of refraction will equal infinity, which means that the re-selling price of such goods is equal to zero or that it is not possible to re-sell them at all. A typical example of this would be the case of services which perish in the moment of their acquisition.

From all this it follows, that as far as refractions are concerned, the best store of value is money, and the worst (which is not a store of value at all) is in goods which cannot be re-sold as e. g. services.

But, as we have said above, not only refractions, but also uncertainties are of importance for the store of value function.

Here the situation becomes more difficult. To say that a good is not affected by uncertainties means no less than that

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[1] The same lack of refraction would hold good for any other good, more or less generally accepted in exchange for others. But such a good would in fact perform monetary functions.

we know with certainty all our future changes in tastes, as far as this good is concerned, and all future changes in its price.

Now, it is true that there are goods in relation to which our tastes are fairly stable; there are also some for which changes of prices may easily be predicted. But we never can say that in connection with any good uncertainty is entirely irrelevant.

The second difficulty about uncertainty is that (unlike the refraction) it is a purely subjective notion. It is possible to lay down some principles for the classification of goods into those more or less liable to changes of data, but this classification may be only a very general one. The difficulty lies in the fact that the changes of data may be of different types (e. g. of tastes, in property and income and of prices in the market) and therefore a good fairly safe from a change of one type may be particularly liable to changes of an other type. On such a basis it is possible for the individuals to form estimates of uncertainty connected with particular goods, but naturally they are bound to differ very considerably in their opinions.

The third complication connected with uncertainty is perhaps the most serious one.

Uncertainty means the possibility of change. Now we know that a change in one point of a preference system necessarily causes (as a result of the interdependence of goods) a fairly wide-spread movement of secondary changes and adjustments. Consequently uncertainty connected with one single good is not so very important, because it may be influenced by uncertainty connected with goods interdependent with it.

Here lies the chief difficulty of attributing uncertainty to particular goods; as we have seen this can be done in a very general way only.

Quite a different situation confronts us as far as price refractions ars concerned. These are specific to particular goods and do not change in consequence of changes that may affect some interdependent goods [1].

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[1] This results from the fact that they are objective data. They may change of course as a result of changes in other data, but this will be done, if at all, in a very round-about way by the influences of tastes on demand, of demand on prices etc. This sort of change we are justified in considering a new change of data for the individual.

There remains yet the question of money in connection with uncertainty. We have said already that no one good is free from uncertainty, it should be made particularly clear that money is not an exception in this respect, although it is in a kind of special position as far as refractions are concerned.

Money makes for safety from one change of data only. This is from the change in tastes, because by keeping money the consumer postpones, so te speak, his decision as to buying of goods. Somebody who is not sure of the stability of his own tastes should keep a comparatively large cash balance and avoid buying other durable goods. But on the other hand durable goods are a much better store of value than money when a rise of prices is expected and the changes of tastes are less likely to occur [1].

In concluding this paragraph we may say that an individual who wishes to find a good which will perform well the store of value function must take into account both uncertainties and refractions connected with it. His choice will be different according to what kind of changes in data he is expecting to occur.

6.

In the whole discussion of the concept of refrected prices which we have carried on up to this point we have limited ourselves to the problems confronting an individual consumer in this connection. There is, however, no doubt that a similar situation may confront not only the consumer, but the entrepreneur as well.

If the entrepreneur controls a commercial firm his situation will be apparently very similar to that of a consumer. He will be confronted with two markets for the good he deals in. These are the wholesale market in which he buys this good and the retail market in which he sells it.

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[1] This statement is in two ways different from Dr Hicks' opinion in this question (*A Suggestion for Simplifying the Theory of Money*, Economica 1935, p. 8). Firstly because it is contended here that uncertainty is met by holding not only money, but also other goods, and secondly because the store of value function is shown to be performed by money in an imperfect way.

But there is a very definite difference between these two cases. It lies in the fact that the good the entrepreneur buys is for him definitely a different good from the one he sells. The material identity of them is of no importance to him. From this it follows, that the differentiation made by the entrepreneur corresponds to the differentiation generally accepted in the market. There are therefore in this case two goods and two prices. The problem of refracted prices cannot arise here because its essence consists in one good having two prices, a situation that may take place only when there is a lack of uniformity in the differentiation of goods.

The whole of our previous reasoning applies therefore to the consumer only.

The difference between the consumer and the entrepreneur in the manner of differentiating goods is only one of the aspects of the essentially different attitude they have towards all economic problems confronting them [1].

The problem of multiple prices for a good materially uniform, which for the consumer takes the form of refracted prices, seems to be of importance to the entrepreneur too; but this question falls outside the scope of the present paper.

7.

Consumer's Equilibrium

Let us turn now to a new question, namely to the problem of the equilibrium position that is reached by the consumer when the refracted prices are recognised.

The assumptions we have made previously about anticipations and uncertainties are not essential in the discussion of this aspect of refracted prices [2]. All we need is, firstly, a level of abstraction admitting differentiations by the buyers and therefore the appearance of price refractions i. e. the usual

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[1] Cf. Hans Mayer, *Untersuchungen zu den Grundgesetzen der Wirtschaftlichen Wertrechnung.* Zeitschrift für Volkswiftschaft und Sozialpolitik 1922, p. 21.

[2] We shall, however, recognise the existence of uncertainty by having money or the cash balance as a separate good in our system of indifference.

imperfect competition level of abstraction; and secondly the possibility of conceiving a deviation from a previously established equilibrium, that is, an actual appearance of changes of data [1]. This will mean that we must pass from the simple static system to the so-called comparative statics.

We may represent the situation of the consumer in a system of indifference diagrams. The most conveniant way of constructing it would be to measure money along the X axis of every diagram, and the good the refraction of which we are interested in along the Y axis. Doing this we must however remember the interdependencies of goods, i. e. that a change in one diagram in the system is likely to be followed by changes in all the other diagrams.

It is possible to avoid using money in this connection and to measure some other good along both the X and the Y axes. The results will be the same. But the procedure we are to adopt in this paragraph seems to be the more conveniant for this reason: viz. that it corresponds to reality where all goods are bought and sold for money and where prices are expressed in money. The consideration of money as a good having an independent utility is fully justified as we recognise the existence of uncertainty in the system [2]. The drawing of indifference curves between money and any other good presents therefore no difficulty.

Let us examine first the circumstances the consumer is confronted with when, as a result of a change of data, he is out of equilibrium and inclined to move towards it. This is represented in fig. 2 where the consumer is at the point P.

We may call the proportion between the market price and the marginal rate of sustitution at P the r a t e o f d e v i a t i o n.

It is the proportion between the rate of deviation from the equilibrium and the rate of refraction that is of relevance here.

The line PA represents the price in a case when there is

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[1] This must be clearly distinguished from the anticipations of changes of data which have already been discussed.

[2] This follows from the previous argument (par. 4 and 5); see also J. R. Hicks, op. cit. Economica, February 1935, for the most recent of similar formulations of this question.

no refraction whatever, i. e. when the rate of refraction is equal to unity. The rate of deviation is in this case obviously greater than unity, and therefore greater than the rate of refraction.

In such a case the equilibrating movement will be performed in a simple way. Our consumer will move along the path *PE* and stop at *E*, which is the tangent point of this path to the highest indifference curve and where the market price is equal to the marginal rate of substitution. The point *E* is therefore an equilibrium point in the old Paretian sense.

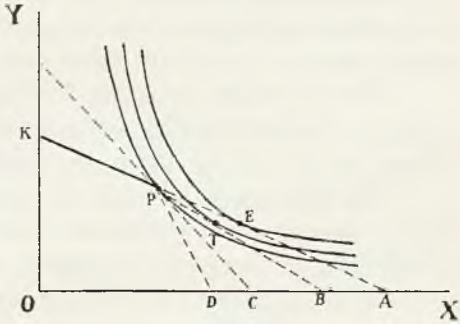


Fig 2

But we have shown already that this is usually impossible. We must have some refraction of price.

Let the refracted price be represented by the line *PB* (fig. 2). In this case the rate of refraction is greater than unity, yet less than the rate of deviation. In order to reach the equilibrium position the individual will move along the path *PT* until he reaches the point *T*, which is the highest point attainable by that path. We shall call it the terminal point. The equilibrium will be established there; but of course a different equilibrium from the previous one.

The rate of refraction may however happen to be still greater than in the last case. It may be for instance equal to the rate of deviation. The refracted price will then be represented by the line *PC* (fig. 2). It is clear that in such a case the point *P* will be the highest point in the individual's preference system that is accessible by paths at his disposal. The terminal point will therefore coincide with *P*. No movement will be performed by the individual in spite of the existing deviation.

The same will be the case if the refraction is greater than the deviation as shown by the line *PD* in fig. 2.

We may express this situation by saying that where a price refraction takes place, the paths the individual is able to follow (in order to reach the equilibrium after a deviation)

are refracted too. This is so when the deviation from the equilibrium has surpassed the existing refraction. If the deviation does not surpass the price refraction the paths will no longer be refracted, they will be locked, and the individual will be not able to move at all i. e. he will remain where he was brought by the deviation.

Let us examine now the situation of an individual who has been in equilibrium but is confronted with a change of data which means a deviation from the equilibrium to him.

Two cases are possible. The individual may be in equilibrium at a terminal point which he reached in the way described above, or he may be at an equilibrium point proper.

The essential difference between these two points is that in the case of the equilibrium point proper the marginal rate of substitution equals the market price, and in the case of the terminal point it equals the refracted price. From this it follows that the individual responds differently to changes of data that may occur and cause deviations from the equilibrium.

The situation when the individual at a terminal point is confronted with a change of data is represented in fig. 3. The marginal rate of substitution is represented there by the tangent line LB. The angle KTB represents the price of Y.

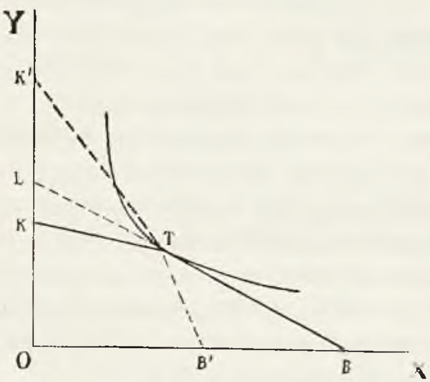


Fig. 3

It is easily seen in fig. 3 that quite a large fall in the price of Y must take place to induce the individual at T to buy Y i. e. to move to the left. This fall of price must be sufficiently large to make the deviation, being its result, greater than the refraction of price. In the diagram (fig. 3) this deviation will be represented by the shift of the price line from the position KTB to some $K'TB'$, its upper part being upwards and to the right from the marginal rate of substitution line LT.

The same applies, of course, not only to changes of price,

but also to other changes of data, which are causes of deviations against Y (i. e. diminishing its price relatively to its marginal rate of substitution).

Just the opposite will be the situation in the case of a deviation in favour of Y. The consumer at T (fig. 3) will be inclined to sell a part of his Y as soon as the price change will be felt.

An individual at an ordinary equilibrium point (as shown in fig. 4) would react in a diametrically opposite way in the case of these changes.

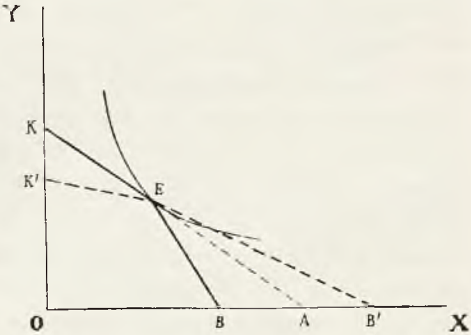


Fig. 4

In fig. 4 KA is the marginal rate of substitution, and KEB the price. A fall in the price of Y will be followed by an immediate adjustment. A rise in the price of Y will have to surpass the price refraction to cause an adjustment. The price line must be shifted to a new position $K'EB'$ of which the lower section EB' must be to the right of the line EA.

In all these examples we have assumed that the rate of refraction remains constant while the prices change. This is a rather rough approximation but it does not affect the essence of the argument.

8.

Pareto's Terminal Points

In the previous paragraph the term "terminal point" was used. This is not a new term in the theory of value. It may be found in Pareto's "Manuel" [1], where it is defined as "un point... au déla duquel les obstacles ne permettent pas à l'individu d'aller". This is a very inadequate definition and leaves the reader in doubt as to what is really meant by it; the more so because those terminal points are scarcely mentionned in the

[1] V. Pareto, *Manuel d'Economie Politique*, ch. III, 62, p. 172.

following parts of the "Manuel" and not mentionned at all in the Appendix.

If, however, Pareto has not given us the exact meaning of this concept we may try to give our own explanation of it, and in fact what has been written in the previous paragraph may be looked upon as an attempt at this [1].

Pareto's system of equilibrium is, as we know, based on very abstract assumptions; the individuals' tastes and the amount of resources are the only data given. To establish an equilibrium one distribution is necessary and sufficient. No changes of data are anticipated. In such a system there is no room for terminal points which as we have shown, arise as a result of refracted prices, and therefore belong to a less abstract system than the Paretian one. But as Pareto has dealt with the most abstract system only, he could not give any precise definition of the concept of terminal points. He only mentioned its existence thus making an excursion into a realistic world, otherwise unconnected with the whole of his system.

What has been said about terminal points in the previous paragraph may be taken as an attempt to give a more precise formulation of the coucept of Paretian terminal points i. e. points of equilibrium other than tangent points of the indifference curve and the line of the market price [2].

9.

To sum up; the main task of this paper has been to find out the relevance of a more realistic concept of the market to the situation of an individual consumer.

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[1] Curiously enough, so far as I know, hardly any attempts at developing this point have been made at all. A. Ossorio in his *Theorie Mathematique de l'Echange*, pp. 333 and 334 touches upon this point, but his comment is short and unsatisfactory.

[2] We have reasoned on a level of abstraction which is implied in the assumptions of imperfect competition. If we go on reasoning at still more realistic levels we shall find more and more data relevant to the determination of the terminal points. These will be e. g. discontinuities, costs of buying and selling and all sorts of frictions. These are left out of the scope of the present paper, which is concerned with the imperfect competition level of abstraction only.

Multiple ways of differentiating goods have been proved to be for him the most important consequence of "imperfections"; the phenomenon called price refraction has been deduced from this and its implications discussed.

These have been shown to be of importance to the individual's conduct as soon as anticipations and uncertainties are recognised. The concurrence of uncertainties and refractions for determining the store-value function of goods has been inquired into in some detail.

The second major implication of the existence of refractions appeared in the discussion of successive equilibria. New equilibrium points were established. They have been called terminal points, they may be conceived as corresponding to the Paretian undefined concept of the same name.

Streszczenie

Jan Drewnowski: Równowaga konsumenta a rynek niedoskonały

1. Praca niniejsza ma za zadanie zbadanie warunków, w jakich znajduje się indywidualny konsument przy przyjęciu założeń t. zw. niedoskonałej konkurencji, to znaczy, w jaki sposób osiągana będzie przez niego pozycja równowagi.

Przejście z doskonałej do niedoskonałej konkurencji polega, według powszechnie przyjętej definicji tych pojęć, na odrzuceniu dwóch założeń, a mianowicie nieskończonej liczby sprzedawców i doskonałej jednolitości produktu. Do tego dodać można przejście z doskonałej pewności do systemu, w którym panuje niepewność.

2. Niedoskonała jednolitość produktu oznacza dla konsumenta możność różniczkowania produktu fizycznie jednakowego według innych cech, niż jego materjalne właściwości. Różniczkowanie to będzie m. in. dokonywane według osoby sprzedawcy. Skutkiem tego cena, którą konsument będzie musiał płacić za każde dobro nabywane u zawodowych sprzedawców, będzie różna od ceny, którą będzie mógł uzyskać odsprzedając sam to samo dobro.

3. Zjawisko powyższe nazwać można zjawiskiem ceny załamanej, a to dlatego, że linja ceny na wykresie krzywych

indyferencyj, obrazująca ten stan rzeczy, będzie załamana w punkcie, w którym znajduje się konsument.

4. Z określonego powyżej zjawiska załamania cen wyprowadzić można pewne jego dalsze konsekwencje. Przedewszystkiem stwierdzić można, że istnienie niepewności wtedy dopiero nabiera większego znaczenia dla postępowania pojedyńczego konsumenta, kiedy uwzględnimy istnienie załamanych cen.

5. Załamanie ceny wraz z niepewnością mają znaczenie dla zagadnienia "przechowywania wartości". Im załamanie ceny jakiegoś dobra jest mniejsze, tem lepiej spełnia ono funkcję "przechowywania wartości". Dla ujęcia jednak całości tego zagadnienia trzeba także uwzględnić niepewności związane z poszczególnemi dobrami, co jest rzeczą z kilku powodów dość trudną do zrobienia.

6. Wszystkie rozważania niniejszej pracy odnoszą się wyłącznie do konsumenta i nie mogą być stosowane do przedsiębiorcy, który w sposób zasadniczo odmienny od konsumenta podchodzi do zagadnień ekonomicznych.

7. Równowaga konsumenta będzie osiągnięta przy uwzględnieniu załamań cen w innych punktach, niż była osiągana bez uwzględnienia załamań. Nowe pozycje równowagi mogą być przedstawione na rysunkach.

8. Te nowe punkty równowagi (osiągane przy uwzględnieniu załamań) mogą być nazwane punktami zatrzymania (points terminaux). Terminu tego używał Pareto dla oznaczenia pozycji równowagi, nie znajdującej się w punkcie styczności linji ceny z krzywą indyferencji, bez wyjaśnienia zresztą, w jaki sposób te punkty są wyznaczane. Całe rozumowanie niniejszej pracy, mające na celu wyznaczenie punktów równowagi konsumenta w niedoskonałej konkurencji, może być uważane za próbę ścisłego zdefinjowania Paretowskich punktów zatrzymania.

JAN WIŚNIEWSKI

NOTE ON SEASONAL VARIATION

The multitude of known methods of isolating the seasonal component might all be considered to deal in fact with a special class of correlation problems. The purpose of correlation methods is to isolate the influence of the factor or factors considered interesting or "principal" from those deemed "random". With the seasonal component the situation is just the same. As we have known since the first works of the Harvard school, each economic time series is a composite of at least four elements: seasonal, trend, cyclical and random. Our purpose is to calculate the dependence of the seasonal component on time (and possibly other variables also), i. e. to determine indices of seasonal variation for each separate month. This is actually a regression function.

The degree of accuracy with which the regression function represents the actual dependence, may be expressed by a measure of the general type

$$R = \sqrt{1 - \frac{s^2}{\sigma^2}} \quad (1)$$ [1]

As we know, special cases of this formula are the correlation coefficient, the correlation ratio and many others. σ is the standard deviation of the dependent variable, while s is the standard deviation of the residuals after the regression formula has been applied. Upon the precise meaning we give to s and σ the interpretation of R depends. In the case of seasonal variation R

[1] Cfr. F. C. Mills, *Statistical Methods Applied to Economics and Business*, New York 1924, p. 373.

should be interpreted as closely analogous with a partial correlation coefficient. As a matter of fact, in all methods aiming at isolating the seasonal component, trend and cyclical variation are first to be eliminated by some means or other. Therefore σ should measure the dispersion of the values observed from values expressing the joint effect of trend and cyclical variation, while s — the dispersion of the values observed from those expressing the joint effect of trend, cyclical and seasonal variation.

Another possibility would be of calculating σ from the arithmetic average of all data and s from this average plus the seasonal component. This possibility has been chosen by Mr. R. von Huhn[1], his measure being

$$\eta = 1 - \frac{s}{\sigma} \tag{2}$$

We cannot consider Huhn's methods as well adapted to the specific purpose of measuring the degree of seasonal adjustment. Suppose the series analysed undergoes violent cyclical changes, while the seasonal component is small but regular i. e. free from random oscillations. In such a case the value of η would be numerically small, thus contradicting its purpose of measuring the degree of seasonal adjustment which in this case could be nearly perfect. Such cases are present in the example given by Mr. von Huhn. His measure would give the limiting value 1 only if by means of seasonal adjustment we were able at once to eliminate not only seasonal but also trend, cyclical and random variation. I think this is really exacting too much. The R measure, on the other hand, gives 1 in the case when after eliminating seasonal variation, trend and cyclical variation only is left, and 0 when seasonal adjustment is simply non-existent.

Now the practical problem — apart from any method of isolating the seasonal component — is to determine such values of the variables as exhibit only the influence of trend and cyclical variation. Such values we shall call normal values. Provided random variation is restricted to oscillations small

[1] *Standard Deviation as a Measure of Seasonal Adjustment* — Journal of the American Statistical Association, March 1933, pp. 70—75.

in amplitude and of a duration not exceeding a few months, the most plausible method of determining "normal" values seems to be the thirteen-months moving average, where the values of the first and last month are given half the weight of the remaining months. Such an average can be assumed to eliminate all of the seasonal component (provided it is additive[1]), and practically all of random variation as defined above. A problem arises, whether this average does not eliminate, in surplus, some of the trend-cyclical variation also. This is strictly impossible only in the case when the true normal values form an arithmetic progression. In other cases some discrepancy may arise. It will be negligible if the series of true normal values is monotonic and can only assume any significant values when a local maximum or minimum lies within the thirteen months period, yet its magnitude will always be very small.

Some methods of estimating this discrepancy are feasible, but they require additional simplifying assumptions. One such assumption is that suggested to me by Mr. Kalecki, namely that the cyclical oscillations about the trend line are expressible in terms of a sine curve. Then, as is readily calculable, the ratio of the deviation of the moving average from the trend line to the deviation of the true normal value from the same line is constant and equal to $\left| \sin\frac{\pi}{p} \right| : \frac{\pi}{p}$ where p means the duration of the business cycle in years. This method would, of course, require additional investigation as to the shape of the trend line. Taking $p = 10$, the correction factor would amount only to $1^1/_2\%$ of the deviation from trend. Another possibility consists in assuming the true normal line within the limits of one year to be expressible in terms of a second degree parabola. In this case the correction would be additive and amount to approximately 3.8 times the second difference of the normal values. Were it not for random fluctuations, the second differences of the moving average and of the true normal values would be the same; in practice, however, it is pretty difficult to ascertain the magnitude of the second differences. As expe-

[1] And moreover, constant: if this component changes from year to year but slightly, it will be also eliminated with accuracy sufficient to our purposes.

riments have shown, the correction rarely exceeds two per cent. of the moving average.

What is the field of application of the measure R? First, it can be used for the purpose of estimating what part of the dispersion of the variable under investigation — after elimination of trend and cyclical variation — is expressible in terms of seasonal fluctuations. For this purpose R^2 seems even better adapted than R, as it measures directly the proportion of variance attributable to the seasonal component [1].

Other applications are: comparison of two series from the same period and adjusted by the same method, comparison of seasonal adjustment, in the same series but in two different periods, comparison of the degree of seasonal adjustment obtained in the same series by two different methods. The method giving a higher value of R, should be considered as superior.

The point last mentioned suggests the possibility of devising a method of seasonal adjustment that would give the maximum value of R. Let x denote the observed values of the variate in question, a — normal values (for the purposes of computation — 13 months moving averages), w — the seasonal component, which is considered constant for all years. We postulate

$$\sum_{1}^{12} w_i = 0 \tag{3}$$

then

$$s^2 = \frac{1}{12n}\left[\sum_{1}^{11}\sum_{i}(x - w_i - a)^2 + \right. \tag{4}$$

$$\left. + \sum_{12}(x - a + w_1 + w_2 + w_3 + \dots w_{11})^2 \right].$$

The normal equations are

[1] This field of application only has been discussed by Simon Kuznets, to whom belongs the priority of using a measure virtually identical with R, as defined above. Cfr. *Seasonal Variations in Industry and Trade*, New York 1933, pp. 33—35.

$$2w_1 + w_2 + w_3 + \dots w_{11} = \frac{1}{n}\left[\sum\nolimits_1 (x-a) - \sum\nolimits_{12} (x-a)\right] \quad (5)$$

$$w_1 + 2w_2 + w_3 + \dots w_{11} = \frac{1}{n}\left[\sum\nolimits_2 (x-a) - \sum\nolimits_{12} (x-a)\right]$$

$$w_1 + w_2 + w_3 + \dots 2w_{11} = \frac{1}{n}\left[\sum\nolimits_{11} (x-a) - \sum\nolimits_{12} (x-a)\right]$$

the solution being

$$w_j = \frac{1}{n}\sum\nolimits_j (x-a) - \frac{1}{12n}\cdot\sum_{1}^{12}\sum\nolimits_i (x-a) \quad (6)$$

where $\sum_i$ signifies a summation extended over the values in the month i in all years.

The method outlined above differs from the method of moving averages in that arithmetic means instead of medians are employed. The use of the median, however, is justifiable if we intend to extrapolate the computed values of the seasonal component; in certain cases the stability of the median is greater than that of the arithmetic mean [1].

It should be noted, that the R measure may be applied to logarithms of the series under investigation, if we are interested in ratios rather than in absolute deviations. In this connexion it is perhaps worth-while observing that the link-relative method of seasonal adjustment is bound to give results nearly identical with the moving average method applied to logarithms, as I have shown elsewhere [2].

After this comparatively simple case let us proceed to a more complex one. It seems worth-while to take first a bird's eye view of the many schemata of the behavior of the seasonal component. Some schema is necessary in order to simplify the multifarious reality and enclose it within the bounds of mathematical formulae.

We shall use the same notations as before, adding to

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[1] Cfr. H. S. Pollard, *On the Relative Stability of the Median and Arithmetic Mean etc.* Annals of Mathematical Statistics, September 1934, pp. 227—262.

[2] Prace Instytutu Badania Konjunktur Gospodarczych i Cen. Warsaw, Nr. 1, 1935. (With a French summary).

them e — the random component and t — time, measured in years. Then by definition

$$x = a + w + e \tag{7}$$

for any given t. We must not assume these magnitudes to be independent. As a matter of fact we may write

$$w_i = f_i(a, t) \tag{8}$$

denoting with i the number of the month (January — 1, ... December — 12) Among the many hypotheses concerning the shape of the function f the simplest seems to be that already analysed, viz.

$$w_i = \text{const.} \qquad i = 1, 2, \dots 12 \tag{9}$$

fo any given month i. The most frequently made is

$$w_i = p_i\, a \qquad i = 1, 2, \dots 12 \tag{10}$$

The first hypothesis may be called that of a constant seasonal component or of additive seasonality, the second one — of a constant seasonal factor or of multiplicative seasonality. While the first is the simplest (containing, as it does, the special case of $w_i = 0$ i. e. of no seasonal variation) and the second the most frequently adopted, they by no means exhaust all possibilities. One of them consists in writing

$$w_i = p_{i,t}\, a \tag{11}$$

i. e. making p vary not only from month to month but also from year to year (progressive seasonality). This was the subject of investigations by King, Crum, Gressens and others. In what follows we shall pursue another path and assume the form of dependence

$$w_i = p_i\, a + q_i \qquad i = 1, 2, \dots 12 \tag{12}$$

For the sake of brevity this case will be referred to as linear seasonality.

Linear seasonality is, of course, a special case of changing seasonality. If we make a comparison of the values of a variate obeying the law of linear seasonality in two different years, then neither their ratio nor their difference will be free from the influence of seasonality. As contrasted with progressive

seasonality, linear seasonality does not aim at expressing the seasonal component as a known function of time. For any two years — however far apart — the magnitude of the seasonal component w will be the same if only the values of a are identical. This contrasts also with the scheme devised by S. Kuznetz (op. cit. ch. X, XI) were it used for estimating each year's seasonal variation.

Now it is a question how to determine the values of the parameters p and q. In accordance with what was said above, these parameters must fulfill the condition that

$$s^2 = \frac{1}{12n}\left[\sum_{1}^{12}\sum_{i}(x - w_i - a)^2\right] \tag{13}$$

be a minimum. n denotes the number of years investigated, $\sum_i$ the sum of data of the i-th month of all years. The shape of w is given by (12). Denoting

$$P_i = 1 + p_i \tag{14}$$

we get

$$s^2 = \frac{1}{12n}\left[\sum_{1}^{12}\sum_{i}(x - P_i\, a - q_i)^2\right]. \tag{15}$$

If the values P_i and q_i for $i = 1, 2, \ldots 12$ were independent between them, the solution of the minimum problem would be no more than an application of classical formulae of the least squares method. The values of the parameters which we should get on this assumption might be denoted by P and q; they are as follows:

$$P'_k = \frac{\sum_k ax - \frac{1}{n}\sum_k a \sum_k x}{\sum_k a^2 - \frac{1}{n}\left(\sum_k a\right)^2} \tag{16}$$

$$q'_k = \frac{1}{n}\left(\sum_k x - P'\sum_k a\right). \tag{17}$$

In fact, however, we must postulate certain bonds between them. We postulate that if during a year the normal value of

the variate remains the same, the sum of the seasonal components for the several months should equal zero. In other words

$$\sum_{1}^{12}(P_i a + q_i) = 12\,a \tag{18}$$

should be an identity in a. This again reduces to

$$\sum_{1}^{12} P_i = 12 \tag{19}$$

$$\sum_{1}^{12} q_i = 0. \tag{20}$$

The magnitude to be minimized is

$$s^2 = \frac{1}{12n}\left\{\sum_{1}^{11}\sum_{i}(x - P_i a - q_i)^2 + \right. \tag{21}$$

$$\left. \sum_{12}\left[x - \left(12 - \sum_{1}^{11} P_i\right) a + \sum_{1}^{11} q_i\right]^2\right\}.$$

For any $i = k$ we get a pair of normal equations, viz.

$$\sum_{k} ax - P_k \sum_{k} a^2 - q_k \sum_{k} a - \sum_{12} ax + \tag{22}$$

$$+ \left(12 - \sum_{1}^{11} P_i\right)\sum_{12} a^2 - \left(\sum_{1}^{11} q_i\right)\sum_{12} a = 0$$

and

$$\sum_{k} x - P_k \sum_{k} a - n q_k - \sum_{12} x + \tag{23}$$

$$+ \left(12 - \sum_{1}^{11} P_i\right)\sum_{12} a - n \sum_{1}^{11} q_i = 0.$$

Thus we have twenty-two linear equations involving twenty-two unknowns, the problem being in theory solved. Moreover, the

determinant in question is symmetrical about its principal diagonal and any one the simplified methods of solution may be applied. Nevertheless, the amount of labor involved is (in the case of monthly data, at least) absolutely prohibitive and therefore we have to look for an approximate method of solution. From (16) and (17) it is evident that for a given k the value of P'_k is independent of q_k and of the choice of the coordinate axes, while q_k depends both on the value of P_k and on the origin. Therefore we shall first determine the value of P_k. Instead of (21) we shall minimize

$$s'^2 = \frac{1}{12n}\left\{\sum_1^{11}\sum_i (x'-P_i a')_2 + \sum_{12}\left[x' - \left(12 - \sum_1^{11} P_i\right)a'\right]^2\right\} \tag{24}$$

where $x' = x - \frac{1}{n}\sum_i x$, $a' = a - \frac{1}{n}\sum_i a$. Differentiating with respect to P_k we obtain one of the eleven normal equations, viz.

$$\sum_k a'x' - P_k \sum_k a'^2 - \sum_{12} a'x' + 12\sum_{12} a'^2 - \left(\sum_1^{11} P_i\right)\left(\sum_{12} a'^2\right) = 0. \tag{25}$$

It is easy to see that

$$P_k = \frac{P_1 \sum_1 a'^2 - \sum_1 a'x' + \sum_k a'x'}{\sum_k a'^2}. \tag{26}$$

Substituting in the normal equation for P_k we get

$$P_1 = \frac{\sum_1 a'x'}{\sum_1 a'^2} + \frac{12 - \sum_1^{12} \frac{\sum_i a'x'}{\sum_i a'^2}}{\sum_1^{12} \frac{1}{\sum_i a'^2}} \cdot \frac{1}{\sum_1 a'^2} \tag{27}$$

$$= P'_1 + \frac{12 - \sum_1^{12} P'_i}{\sum_1^{12} \frac{1}{\sum_1 a'^2}} \cdot \frac{1}{\sum_1 a'^2}$$

and analogically for any other P_k.

The determination of q is reduced to the problem of a constant seasonal component the role of normal value being played by Pa. The problem is already solved (supra); in fact, we get

$$q_k = \frac{1}{n}\left(\sum_k x - P_k \sum_k a\right) - \frac{1}{12n}\sum_1^{12}\left(\sum_i x - P_i \sum_i a\right) \quad (28)$$

The formulae (27) and (28) should be regarded as good approximations to the "best" values of P and q, permitting the isolation of the seasonal factor within the bounds of linear seasonality. An empirical proof of this contention is given by the fact that while the least squares condition is not simultaneously fulfilled for the P's and the q's, yet the second term of (28), which might be termed corrective, is usually very small. The writer has also performed some trial computations (for quarterly data), which gave almost perfect coincidence with the results of the two methods.

If we wish — as we usually do —to apply the calculated coefficients of linear seasonality for the purpose of "eliminating" seasonality, i. e. of finding the normal value (containing the random impurity, as it were), given the value observed, we may make use of the equation

$$a' = \frac{x - q}{P} \quad (29)$$

where a' denotes "calculated" normal values, not being, of course, identical with a' of formula (24).

Some doubt may here arise, whether we should not compute for this purpose separate equations of regression of a on x. Notwithstanding the prima facie plausibility of such a postulate, much can be said against it. We intuitively demand that there be a certain reversibility in the relationship between raw and adjusted data. Suppose we know the true normal values, then the values a' obtained from (29) for successive months will oscillate around the true values a. If, on the other hand, we should calculate values a'' from a regression equations of a on x, these values would tend, for successive months, to be consistently too great or too small in comparison with a, owing to the fact that the correlation between a and x is not perfect.

If we tried to introduce conditions such as (19) and (20), this would probably reduce both forms of regression lines to virtual identity, not to speak of difficulties in calculation, which are even theoretically insoluble.

The writer has experimented extensively with the application of the linear scheme of seasonality and in many cases it has proved very useful. One should not, however, expect from it too much, as it cannot explain such changes in seasonality as might be termed structural. It should be also understood that we by no means postulate that the regression of x on a is always linear. "Linear seasonality", along with other investigations in the field of changing seasonality, should be regarded as a second step only towards a more efficient description of seasonal phenomena, the first one being research in constant seasonality, whether additive or multiplicative.

Streszczenie

Jan Wiśniewski: Sezonowość linjowa

Oznaczmy przez w wartość składnika sezonowego, przez a normalną wartość zmiennej [1], przez x zaobserwowaną wartość zmiennej, wreszcie przez e wartość składnika przypadkowego. Wtedy tożsamościowo

$$x = a + w + e \tag{1}$$

dla dowolnego momentu. Hipotezie stałej sezonowości addytywnej odpowiada zależność

$$w_i = \text{const.} \qquad i = 1, 2, \dots 12 \tag{2}$$

dla danego miesiąca i. Hipotezie stałej sezonowości multyplikatywnej odpowiada zależność

$$w_i = p_i a \qquad i = 1, 2, \dots 12 \tag{3}$$

Spośród wielu innych możliwości najprostszą będzie hipoteza

[1] Jako wartość tę będziemy przyjmowali w praktyce średnią ruchomą 13-miesięczną.

linjowej zależności między w i a

$$w_i = p_i a + q_i \quad i = 1, 2, \ldots 12 \tag{4}$$

Hipotezą tą zajmiemy się obecnie szerzej. Dla krótkości będziemy mówili o sezonowości linjowej.

Jak wyznaczyć wartość parametrów p i q? Oznaczając $P_i = 1 + P_i$ postulujemy

$$s^2 = \frac{1}{12_n} \left[\sum_{12}^{1} \sum_i (x - P^i a - q_i)^2 \right] = \text{minimum}. \tag{6}$$

Gdyby wartości P_i i q_i dla i = 1, 2, ... 12 były między sobą niezależne, rozwiązanie zagadnienia otrzymania minimum na (6) byłoby zupełnie banalne. Osiągnięte przy tem założeniu wartości parametrów oznaczymy przez P' i q'; przedstawiają się one następująco:

$$P'_k = \frac{\sum_k a x - \frac{1}{n} \sum_k a \sum_k x}{\sum_k a^2 - \frac{1}{n}\left(\sum_k a\right)^2} \tag{7}$$

$$q'_k = \frac{1}{n}\left(\sum_k x - P' \sum_k a\right). \tag{8}$$

W rzeczywistości jednak musimy postulować pewne związki między parametrami P i q. Zakładamy mianowicie, że jeśli na przestrzeni roku wartość normalna zmiennej pozostaje bez zmiany, to suma składników sezonowych dla poszczególnych miesięcy powinna się równać zeru, co znowu sprowadza się do dwóch warunków:

$$\sum_1^{12} P_i = 12 \tag{10}$$

$$\sum_1^{12} q_i = 0. \tag{11}$$

Niestety, po uwzględnieniu warunków (10) i (11) rozwiązanie układu dwudziestu dwóch równań normalnych staje się bardzo skomplikowane, tak iż trzeba się uciec do metod przybliżonych. Określenie parametrów rozłożymy na dwa etapy.

Ze wzorów (7) i (8) okazuje się, że dla danego k wartość P' nie zależy od wartości q i od wyboru osi współrzędnych, gdy tymczasem wartość q zależy zarówno od P jak i od położenia początku układu. Wobec tego określimy najpierw P_1 i analogicznie pozostałe P_i.

$$P_1 = P'_1 + \frac{12 - \sum_1^{12} P'_1}{\sum_1^{12} \frac{1}{\sum_1 a'^2}} \cdot \frac{1}{\sum_1 a'^2}. \tag{15}$$

Teraz oznaczymy q. Sprowadza się to do zagadnienia sezonowości stałej addytywnej, przyczem jednak zamiast a rolę wartości normalnych będą spełniały wartości Pa. Zagadnienie to zostało rozwiązane w Pracach Instytutu Badania Konjunktur Gospodarczych i Cen, zesz. 1/35[1] (wzór 12). Otrzymujemy

$$q_k = \frac{1}{n}\left(\sum_k x - P'_k \sum_k a\right) - \frac{1}{12n} \sum_1^{12} \left(\sum_i x - P_i \sum_i a\right). \tag{16}$$

Eliminowanie sezonowości odbywa się w ramach hipotezy linjowej przez zastosowanie wzoru

$$a = \frac{x - q}{P} \tag{17}$$

Nasuwa się tutaj pewna subtelna kwestja: czy dla eliminowania sezonowości nie należałoby stosować innych równań, niż dla opisu sezonowości, podobnie jak inne jest z reguły równanie regresji X względem Y niż Y względem X. Sądzimy, że na pytanie to należy odpowiedzieć przecząco ze względów ściśle związanych z naturą szeregów rozwojowych. Poszczególne obserwacje nie są losowane kolejno dla najrozmaitszych wartości a, lecz przeciwnie dla wartości a niezbyt różniących się między sobą (choć może nam nieznanych). Wskutek tego w obrębie roku następuje wyrównanie odchyleń wartości x od ich równań regresji względem a, gdy tymczasem podobne wyrównanie odchyleń wartości a od ich równań regresji względem x bynajmniej nie następuje i przez to dla poszczególnych lat mielibyśmy od-

[1] Czytelnikowi polskiemu proponujemy zapoznać się z całością zamieszczonego tam artykułu, jako ze wstępem do niniejszej pracy.

czytane z równań regresji wartości *a* systematycznie za niskie lub za wysokie. Możnaby wprawdzie zaradzić temu przez postawienie postulatu jak w (9), lecz wprowadzenie odpowiednich poprawek prawie zidentyfikowałoby oba typy linij regresji.

Powyżej nakreślone metody znalazły już zastosowanie w obliczeniach Instytutu Badania Konjunktur Gospodarczych i Cen.

ABSTRACTS FROM CONTRIBUTIONS TO ECONOMICS PUBLISHED IN POLISH IN THE PERIOD 1934—1936

Dr. Adam Krzyżanowski, Professor in the Jagiellonian University, Cracow, *Dolar i złoty. Towarzystwo Ekonomiczne w Krakowie. Kraków 1936. (The Dollar and the Złoty. Economic Society in Cracow, 243 pages)* [1].

At the close of the World War, the currency of practically all nations taking part in the war, underwent a heavy depreciation. Stabilization action which soon followed, was based on credits offered for the most part by the United States; international credit also made it possible for Germany to pay her Reparations agreed on in the Versailles Treaty. These great international payments were possible only thanks to credit inflation in the United States. That inflation swept over Europe through stabilization and Reparations credits.

A result of credit inflation was the avoidance — as it turned out — of a temporary reduction of prices, which was only to be expected after the completion of the war; a policy

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[1] This study forms a part of a collection of economic papers published under the same general title. The collection contains, besides "The Dollar and the Zloty", the following studies: The demographic features of the depression. The gold stock of the Bank Polski. The treasury and the currency. The dispute about the currency policy. The internal loan. The theory and the policy of saving. The contemporary spirit of economy. Hunger in the communistic country. The surplus of grain in the United States. Winding up of the business cycle in the United States. Credit policy of the government. Real and monetary savings. Forecasts which have not been fulfilled. Plan of financial and economic reconstruction agreement.

The editors are publishing with the author's agreement an abstract of the study *The Dollar and the Zloty* as the most interesting for foreign readers.

of cheap money also brought about a rise in exchange quotations, and speculation in stocks. Only the prices of agricultural products were unable to maintain themselves on the war-time level. The breaking up of the stock exchange rates in New York in the fall of 1929 closes the period of credit inflation. Europe withdraws her capitals invested in stock speculation in New York, and these capitals did not form the basis for new credits owing to lack of confidence.

There follows a period of hoarding in the United States as well as in Europe, which is even made more severe by the bankruptcies of great banks. The ebbing away of gold from England, whose capitals are frozen up in Germany, Austria, and Hungary, forces the Bank of England to abandon the gold clause for banknotes, a step which a series of countries followed.

Soon afterwards all payments on the part of Germany for Reparations cease completely — as do all payments to the United States by winning countries on war debts.

One result of the repudiation of international political debts and payments resulting from them, was a small rise in the stock market, which was stopped, however, by the currency policy of the United States. The world production of gold reached about 600 millions of dollars in 1933, and hoarding reached almost this amount. The fall of the dollar and its provisory stabilization at a level of about 41% lower than the gold clause complete the series of events.

The Polish *Zloty* had the character of gold check currency, since the Bank of Poland sells only checks redeemable in gold and at a fixed rate. The rate of checks is non-elastic in comparison with the gold rate and that of banknotes redeemable in gold, which may oscillate within cartain limits. The hoarding panic appeared here in the rise of the price of gold money and in the arbitrages resulting from it. As a result the gold supply of the Bank of Poland was diminished while the supply of gold in the hands of the public increased by about the same amount. Besides the banknotes of the Bank of Poland, dollar banknotes also circulated in Poland, a double currency left from the times of the inflation of the mark, the introduction of the zloty not pushing the dollar out of circulation since in the following year the zloty fell. In a still

greater degree, the dollar was used as a measure of value, especially in the case of long-term credits, the payments themselves being effected in zlotys. Not rare, either, were bank deposits made in dollars, as well as insurance agreements based on dollars. Dollar banknotes and money were in circulation and were also hoarded. Confidence in the stability of the dollar was evidenced in the difference in the interest, offered on dollar and on zloty deposits. The owners of dollar deposits were satisfied with a interest 2—4% lower than that paid on deposits in zlotys.

According to law in Poland, it is permissible to make agreements calling for payment in foreign currency, and to make payments in that currency; after the devaluation of the dollar it was decreed to apply the American law which eliminated the gold clause. All who had claims balanced in gold dollars lost on this. A new Polish law forbids saving banks and some other financial institutions to accept dollar deposits. Besides losses resulting from the devaluation of the dollar, creditors in Poland also lost on the fall of the Danzig gulden in the year 1935. But business conditions in the years before the war are better than after the war. The withdrawal of capitals from production does not at present affect the credit market, for these capitals are mainly hoarded. Poland after the war lived through five waves of confidence in the currency, and after each of them, hoarding returned. The first in the year 1918 ended with the beginning of the inflation of the mark, the second in 1921 was called forth by the decision to balance the budget and stabilize the currency, the attempt at that time, however, giving no results. The stabilization of the mark and introduction of the złoty took place towards the end of 1924. Conficende was evidenced on the stock market by an increased sale of hoarded dollars and other currency; this process stops, however, in the middle of 1925, and in July the Bank of Poland is forced to raise the price of the dollar above the normal. The next confidence wave appeared in the year 1926, and this time lasted until the outbreak of the international economic crisis in 1929, confidence in the złoty being strengthened by the successful loan transactions. As a measure of confidence we may take the traffic in gold: its increase is

evidence of a growth of anxiety. After the year 1929, the gold turn-over increases and in the four year period 1931—1934, the import of gold in millions of zloty's amounts to 32, 147, 95, 52, and the export about 1, 234, 88, 5. The fact is characteristic that in the whole four year period the import of gold is almost equal to the export, which permits the assumption that hoarded gold in Poland comes from the Bank of Poland. Stress on the Bank of Poland was especially strong in the year 1932, when there were even suggestions to make the zloty rate more elastic. The government decided to keep the present rate of the złoty and allow unhampered traffic in foreign drafts and gold. In Poland there began to accumulate bank drafts from neighbouring countries where an enforced bank draft economy had been introduced, which together with the already possible fall of the dollar strengthened confidence in the zloty. Budget investments also added to the increase of confidence. The threat of the devaluation of the dollar and its fulfillment in the first half of 1933 led practically all over the world to the hoarding of gold; in Poland the fall of the dollar brought about an unusually strong sale of hoarded dollars and gold, and with it the liquidation of double currency in Poland. As a result of the fall of the dollar many losses were suffered in Poland, but the cessation of hoarding compensated for them. In the year 1935 we note the next wave of distrust. The death of Marshall Piłsudski as well as certain events abroad strengthened pessimism.

The circulation of dollars and obligations in dollars is constantly smaller in the period of years directly preceding the fall of the dollar. The very possibility of the fall of the dollar inclined many to the exchange of their dollars and dollar obligations to avoid losses. After the fall of the dollar, this process was hastened. Hoarded dol ars were of course let into circulation and changed. The owners of dollars could buy zlotys, could buy gold for hoarding, or could invest sums received in stocks and loans. They also could place their capitals in the Post Saving Bank Office or in private savings banks, real estate, land, and building projects. The fall of the dollar did not bring about in Poland any increase in the demand for gold; the export of capitals was at that time very limited (being directed mainly

to Palestine in connection with the emigration of Jews to that country). Industrial investments did not increase greatly either.

Renewed circulation of dollars and gold was achieved in part by the building of dwellings, and also took the form of increased złoty deposits in savings banks. In a very great measure it took the form of repatriation of the Polish Stabilization Loan. Owners of Stabilization Loan obligations, although made in dollars, did not lose through the devaluation of the dollar, for they could demand payment according to their choice, not only in gold dollars, but also in gold French or Swiss francs, and in gold Dutch guldens. In such conditions the stabilization loan held great attractions and more than $^1/_3$ of the whole loan is at the present in the country. In the course of two years after the devaluation of the dollar, the amount of stabilization loan obligations at home was increased to about 16 million dollars of the normal value, which means export abroad of about 10 million dollars. The total export of dollars from Poland can be estimated at about 30 some millions of dollars, of which the greater part was exported through the Bank of Poland.

The investment of previously hoarded dollars in the Stabilization Loan, and quite frequently in złotys in gold is evidence that the confidence was not great; hoarding appeared again already in the second quarter of 1935.

The purchase of dollars by the Bank of Poland hindered the fall of money circulation, but did not effect its increase, since the number of dollars offered for sale was not great and the reduction of prices upset the demand for money.

The devaluation and sale of hoarded dollars supplied the Bank of Poland with a considerable supply of foreign bank drafts which permitted the payment of about 100 million zlotys of foreign credit without hurt to its assets; a similar situation was created for the Bank of Poland by the devaluation of the złoty in 1925, with this difference, that at that time the bank drafts came from an export surplus, while now they come from an export of money reserves. The payment of this loan relieves the balance of payments of the quota of the interest on it; the burden of other debts and their services were also lessened. The repatriation of the stabilization loan means that the interest paid abroad remains in the country. The fall in the rate of the

dollar strengthened the rate of national loans and permitted the Treasury to place easily in the market about 650 million of new papers in the form of national loans and treasury bills. The losses of the owners of dollars the author estimates at about 40 million zl. The government buying up dollar obligations of the Post Office Saving Bank added about 30 million zl. in payment. Persons who insured themselves in dollars have at present a lowered sum of insurance, altogether amounting to about 160 million zl. The balance sheets of financial institutions also show losses.

The influence of the devaluation of the dollar on industrial production in Poland was small, building of dwellings only became more lively together with industries connected with it. Among the negative results of the fall of the dollar the author ncludes a falling-off of international economic traffic, as well as hoarding, which appeared in practically the whole world besides Poland.

The devaluation of the dollar strengthened the stability of the złoty, diminished, however, at the time the advantages resulting from such stability. The złoty is fixed only in relation to a few currencies, it is kept stable, however, by import limitations and by high passport fees. In this state of things the stability of the currency ceases to be a strong aid to prosperity and in this way loses its proper *raison d'être*.

The sphere of the circulation of the złoty was wider in Poland than that of the dollar, and for that reason the improvement of business conditions after the fall of złoty in 1925 reached limits considerably broader in comparison with the favourable results of the fall of the dollar. The favourable consequences of the fall of the dollar were temporary. Improvement lasted through the year 1933—1934 and ended at the beginning of the year 1935, since the dollar reserves were exhausted. The situation of the Treasury, temporarily improved as a result of the appearance on the market of hoarded dollars, became worse again, which forced decision either to limit the budget or to cover expenses by inflation. Today again, as before the devaluation of the dollar, we may have to choose between devaluation without inflation, or the maintaining of the rate of the złoty by

way of government control over the exchange of foreign currencies [1].

Dr. Adam Heydel, *Teorja dochodu społecznego. (The Theory of National Dividend). Kraków 1935, reprinted from "Ekonomista", pp. 34.*

In spite of the almost general agreement, that national dividend embraces all net incomes of individuals, its relation to what is called the gross income of society does not appear to be quite clear. There are indeed authors who seem not to discern between these two quantities. Among the classics, J. B. Say was of opinion that "Le revenu d'une nation est égal à la valeur brute et totale de tous ses produits... le produit brut de la société est la même chose que son produit net". The same idea is expressed by a number of contemporary economists: Charles Rist considers that the annual dividend of a nation is its annual production. None of these quantities can be larger or smaller than the other one, as they mutually cover each other [2]. Similar views have been expressed by one of the Polish economists. They are based mainly on the following assumption: national dividend corresponds to the sum of wages, profits and rents, any price can be resolved into wages, profits and rents. The sum of these components of the national dividend is equal to the sum of the prices of goods. In consequence the sum of the prices of all goods produced annually (the gross income) represents the sum of the national dividend. The author of the study under consideration is convinced that this reasoning is based on an erroneous analysis. Its errors are caused by certain difficulties in the exposition of the facts of production and of the processes of formation of the national dividend. The author proposes another approach to these problems which seems to him to facilitate the right solution.

The author begins with the resolution of the price of a commodity into its components. He takes as the point of departure a consumption commodity with the value 100. Chart I illustrates the process of the formation of the price in the course of production.

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[1] The later alternative has been adopted in April 1936. (Editors).

[2] Gide and Rist, *Histoire des doctrines économiques.* Vol. I, Bk II, ch. I.

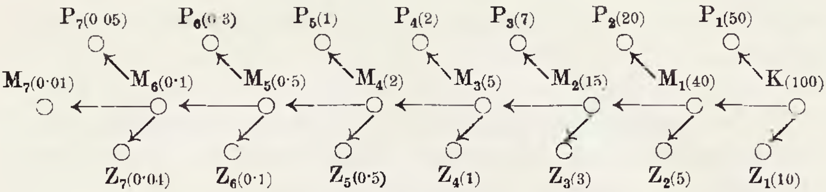


Chart I

K is the consumptive good, M_1—M_7 half finished goods, machinery, rough materials, P_1—P_7, wages, Z_1—Z_7 profits and rents. The price of the good is resolved into these three elements at each stage of production in various relations. The numerical relations on the chart have been chosen arbitrarily. At a very remote stage of production the price will be resolved almost completely into wages and profits. The chart shows: (1) that the price of the good is resolved into its components successively in time, (2) that at each stage of production there remains besides the components *Z* and *P* an unresolved element *M*.

The same analysis may be adopted for any sum of any consumptive good (*K*). The author calculates the numerical results of the figures in chart I. The results are as follows:
1) The sum of the prices of unfinished goods (M_1—M_7) 62.61
2) The sum of wages (P_1—P_7) 80.35
3) The sum of profits and rents 19.64
To discover the meaning of all these numerical results for the problem of national dividend it is necessary to analyse the process of production not only in its singular chain, but in its continuity.

Chart II shows the process of production in its continuity. The vertical axis corresponds to the different stages of production, the horizontal one, to time. The consumptive goods *K* in the time unit *h* result from the adoption of the different means of production which was begun in stage 7 in time *a*. The diagonal row of figures corresponds then to Chart I. To get the same amount of *K* in time unit *i*, it is necessary to start production in time-unit *b* and so on. If all the *K* appearing in time units (*h* ... *o*) are to be equal, the amounts of the different means of production at the same stages must also be equal.

In consequence $M_7 Z_7 P_7 (a) + M_6 Z_6 P_6 (b) + \ldots + M_1 Z_1 P_1 (g)$ must be equal to $M_7 Z_7 P_7 (h) + \ldots + M_1 Z_1 P_1 (h)$.

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*
M_7 Z P	M_7 Z P	M_7 Z P	M_7 Z P	M_7 Z P	M_7 Z P	M_7 Z P	**M_7 Z P**							
	M_6 Z P	M_6 Z P	M_6 Z P	M_6 Z P	M_6 Z P	M_6 Z P	**M_6 Z P**	M_6 Z P						
		M_5 Z P	M_5 Z P	M_5 Z P	M_5 Z P	M_5 Z P	**M_5 Z P**	M_5 Z P	M_5 Z P					
			M_4 Z P	M_4 Z P	M_4 Z P	M_4 Z P	**M_4 Z P**	M_4 Z P	M_4 Z P	M_4 Z P				
				M_3 Z P	M_3 Z P	M_3 Z P	**M_3 Z P**	M_3 Z P	M_3 Z P	M_3 Z P	M_3 Z P			
					M_2 Z P	M_2 Z P	**M_2 Z P**	M_2 Z P	M_2 Z P	M_2 Z P	M_2 Z P	M_2 Z P		
						M_1 Z P	**M_1 Z P**	M_1 Z P	M_1 Z P	M_1 Z P	M_1 Z P	M_1 Z P	M_1 Z P	
							K	K	K	K	K	K	K	K
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*

Chart II

In other words the value of all the half finished goods (M) necessary for the production of K must be equal to the value of all M produced simultaneously with K. The value of $M_7(h) \dots M_1(h)$ forms together with the value of K the so-called gross income. If we adopt the figures from chart I, this value would be $62.61 + 100 = 162.61$. The goods $M_7 \dots M_1$ must however remain unaltered if the consumptive goods are to appear in the same amount in the following periods ($i \dots o$). The only part which can be consumed without any decrease of the future income is K (h). It represents consequently the net national dividend and its value covers exactly the sum of wages, profits and rents which are paid out during the same period (h)[1].

[1] The sum of wages, profits and rents on Chart I was equal to $80{\cdot}35 + 19{\cdot}64 = 99{\cdot}99$. The difference of 0·01 results from the abruption of resolving the price of K at stage 7 ($M_7 = 0{\cdot}01$).

After a discussion showing the turnover of the produced value and dealing with the problem of individual and national net incomes and costs, the author passes to the examination of the assumptions on which he based his demonstration of the process of production and of the formation of national dividend.

He analyses : (1) the notion of a "stage of production" (2) the notion of the "period of production". As to the first of these problems he points out the difficulties connected with the division of the technieal process of production into different stages. Technically the good is undergoing continuous change and the changes in the value of the good must be correspondent. We can as easily consider as one stage of production the whole of the process by which pig iron is changed into a finished motor-car, as, in contrast, the passing of a part of the motor from the hands of one labourer to another. There is no criterion according to which we could choose one of these two series of changes as a basis of the division of the whole process of production into particular stages.

Another external mark which might serve us as a basis for the division of production into stages is the exchange of the good, An act of exchange is habitually bound with a different technical method. (Corn passing from the hands of the farmer into the hands of the miller, and flour passing from the hands of the latter into hands of the baker). The fact of the vertical concentration of production decreases the number of exchanges and in the same way the number of stages.

This change is nevertheless, counterbalanced by an increase in the stocks of the half-finished goods at any of the stages. The reverse should be said of the concentration. Taking this into account we can consider "one stage of production" to be the time during which the good remains in the hands of one entrepreneur. This is of course a simplification and the analysis is the more exact, the greater the number of such stages we are able to differentiate during the course of production. But as the author later proves, the number of stages affects only the exactness of the calculation and not its result.

As concerns the much discussed problem of the period of production the author deals with the following points: (I) whether it is admissible to represent on a diagram the production of

a good at different stages of production divided into equal periods of production. We know that in reality the different stages do not last equally long. (II) whether we can cumulate on the same diagram the production of different goods, in spite of the different time required for their production.

The result of the analysis of point I can be shown on Chart III representing the production of a good with different periods of production at each of the stages of production, — at stage *a* the period of production lasts one month, at stage *b* one week, at stage *c* two weeks. The dotted lines indicate these different technical periods.



Chart III

These differences do not change anything in our score. Two possibilities appear: (1) the produced half-finished product passes from stage *a* to stage *b* once a month. It is also technically further advanced once a month. The factories on the stage *b* do work only one week in a month. The same is repeated at the passage of the product to stage *c*. The factories work here two weeks in a month. As a result the product appears once a month which corresponds to the longest of all the periods of production at the various stages of production. (2) The second possibility is this, that the differences of the periods of production are equalized by the different amounts of the product at each stage. In our case there will be more produced at stage *a* than at the shorter stages, so as to keep all factories running.

II. This last solution gives us also the answer to the question concerning the treatment of different goods on one single diagram. Goods whose production takes more time must be collected into bigger stocks, but they will appear simultaneously with short-run goods.

This solution enables us to compare the production of goods whose per ods of production may be very different and also to compare for the same time incomes flowing from the production of these goods.

The solution of these particular problems throws also

some sight on the general problem of the "period of production". It shows that the point of interest is never the absolute length of time of the period, but the relation of the quantity of goods per unit of the time of labour. The increase of this quantity is the shortening, the decrease is the lengthening, of the period of production.

Its very beginning is without interest, as the quantities (and values) at these very remote stages decrease to infinitesimal magnitudes.

In consequence the value of all the means of production adopted for the production of consumptive goods K of a given value, or (what amounts to the same thing in a static economy), the value of all means of production produced simultaneously with K, can be calculated or at least estimated if some conditions are fulfilled.

The author passes to the examination of this problem which is demonstrated on Charts IV and V.

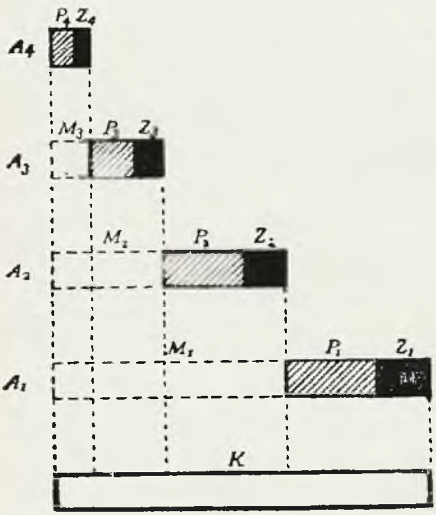


Chart IV

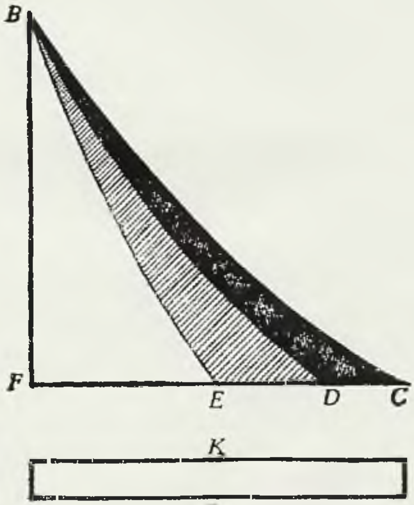


Chart V

Both these charts are constructed on the following basis: on the horizontal axis is measured the value of the products at each stage of production; the vertical axis shows the number of stages of production.

The following equations are derived from this chart:

$$M_1 + P_1 + Z_1 = K$$
$$M_2 + P_2 + Z_2 = M_1$$
$$M_n + P_n + Z_n = M_{n-1}$$
$$M_i + P_i + Z_i = A_i, \quad (\text{for } i = 1, \dots n)$$
$$A_i = M_{i-1} \quad (\text{for } i = 2, \dots n) \quad (1)$$

An exact calculation of the value of all A_i (the surface $B\ C\ F$ on Chart V) is possible under the following conditions: (1) that we know the value of the consumptive goods K, (2) that we know the number of the stages A_i. The second condition does not appear in reality. But even if the number of stages is not known, it is possible to estimate the limite of the value of M, P, and Z. If besides the value of the consumptive goods we know the relative value of M in relation to the value of other means of production at a deliberate stage of production, it is

$$\frac{M_i}{A_i} = m_i$$

The estimation of the total value of M, P and Z can then be obtained by the following method of calculation:

As we know that the relation

$$m_i = \frac{M_i}{A_i} \qquad (2)$$

has a constant value m, or $m_i = m =$ constans, we know then that according to (1)

$$m = m_i = \frac{M_i}{M_{i-1}} = \frac{M_1}{K} \qquad (3)$$

$$\text{or } M_i = m\ M_{i-1} \quad (\text{for } i = 2,\ 3 \dots n)$$

Hence we get

$$M_3 = M_2\, m = M_1\, m^2\ M_2 = M_1\, m$$
$$M_n = M_1\, m^{n-1}$$

Consequently

$$\sum_{i=1}^{n} M_i = M_1 + M_2 + \dots + M_n = M_1 \quad (1 + m + m^2 + \dots m^{n-1}) =$$

$$= M_1 \frac{1 - m^n}{1 - m}$$

From (3) is derived $M_1 = K_{m1}$ therefore

$$\sum_{i=1}^{i=n} M_i = K \cdot m \cdot \frac{1-m^n}{1-m} < \frac{Km}{1-m}$$

We see as a result that even if we do not know the number of stages of production r we can get the desired estimation of the value of $\sum_{i=1}^{n} M_i$. In our particular case this value is smaller than $\frac{Km}{1-m}$

An analogous calculation enables us to estimate ΣM_i even if we do not know the number of the stages of production or the quotient m_i, granted that we know that m_i is smaller than a certain constant number g, which is smaller than one[1].

if it is

$$m_i \leqslant g < 1 \tag{4}$$

For we have then

$$\frac{M_1}{K} \leqslant g, \text{ or } M_1 \leqslant gk \tag{5}$$

And according to (2) and (1)

$$M_i = m_i M_{i-1}$$

Therefore according to (4)

$$M_i \leqslant g M_{i-1} \tag{6}$$

On the basis of (5) and (6) we get

$$M_1 \leqslant g K, M_2 \leqslant g M_1 \leqslant g^2 K$$

Generally

$$M_i \leqslant g^i K$$

Consequently

$$\sum_{i=1}^{n} M_i = M_1 + M_2 + \ldots + M_n \leqslant g k + g^2 k + \ldots + g^n k$$

This means that

$$\sum_{i=1}^{n} M_i \leqslant g k \frac{1-g^n}{1-g}$$

[1] If for instance M_i never is bigger than 0.6 A_i then $m_i \leqslant 0.6$ and we can admit that $g = 0.6$.

and

$$\sum_{i=1}^{n} M_i < g\,k \frac{1}{1-g}$$

Which inequality gives the desired estimation. If for instance for all i the value M_i never exceeds 0.95 A_i we can admit $g = 0.95$ and admitting $K = 100$, we get

$$\sum_{i=1}^{n} M_i < 0.95 \,.\, 100 \frac{1}{1-0.95} = 1900.$$

The author drows the attention of the reader to the fact that the whole of this analyzis is true only for strictly stationary economs. It seems nevertheless that it could serve as a point of departure for studying dynamic conditions.

Dr Jerzy W. Massalski, *Udział robocizny w kosztach produkcji węgla. Polska Akademja Umiejętności. Wydawnictwa śląskie, Prace Ekonomiczne nr 1. Kraków 1935. (The Share of Labour in the Costs of the Production of Coal. The Polish Academy of Sciences and Letters, 46 pages).*

I. The costs of production *(S)*, calculated on the basis of the expenditures of a given enterprise, as shown on its books, are the mathematical function of the quantity produced *(T)*, $S = f\ (T)$.

The object of our studies is a coal mine in the Silesian territory which we have designated by the letter "W". It operates under comparatively good conditions, both natural and technical, and is well organized. The costs of this mine are free from any abnormal deviations, and in the period taken here under consideration, no abnormal accidental factors influenced them in any way. Comparison with the costs of production in other mines shows that the above mentioned mine "W" may be considered as statistically representative.

From the point of view of dependence on the amount of production, the sum of the costs of the mine "W" is composed of invariable and degressively variable. It amounts to 817.800 zl. in the production of 40.000 tons, for 60.000 tons — 955.200 zl., for 80.000 tons — 1.083, 200 zl. and for 100.000 tons — 1,200.900.

The invariable portion of the sum of the costs within the limits of 40 — 100.000 tons, amounts to 532.200 zł., the remainder composes the variable portion.

Dividing the sum of the costs by the amount produced we get the average cost of 1 ton mined. This is expressed by the number 20,44 zł. for 40.000 tons, 15,92 zl for 60.000 tons, 13,54 zl. for 80.000 tons, and 12,01 zł. for 100.000 tons.

The sum of the costs of production we divide for further analysis into a series of technical groups, of which the largest and most important is labour. Closer examination of this group of costs composes the subject of the following paper.

The costs of labour in mining 40.000 tons amount to 351.800 zł., that is, 8,79 zl., for 1 ton mined, for 60.000 tons 438.600 zł., that is, 7,31 zł., for 1 ton, for 80.000 tons 525.300 zl., that is, 6,57 zł. for 1 ton, for 100.000 tons 604.000 zl., that is 6,04 zl. for one ton mined.

In the sum of the cost of labour one may also differentiate between variable and invariable elements. The invariable portion of the sum of the costs of labour amounts to 167.000 tons.

The costs of labour nclude salaries and social welfare and sickness insurance, etc. The latter amount to more than 25% of the costs of labour.

Labour comprises 48.52% of the entire costs of production, amounting to 13,54 zl. a ton.

Other technical groups of the costs of mining coal comprise a smaller percentage of the entire costs and are composed also of variable and invariable elements.

The average price of coal in the period under consideration, calculated on the basis of data supplied by the Institute for the Study of Economic Business Cycles and Prices in Warsaw, amounts to 16,30 zl. a ton. At this price the "W" mine works at a profit, since production is greater than 58.000 tons a month. Bolow this amount of production the costs are greater than the average attainable price on the market, both at home and abroad.

II. The oscillations of the costs of the production of coal in the years 1928—1932 we shall examine: 1) as an average for all Poland in their most important constituents, that is, the cost of labour, administration, wood and coal, and 2) in the costs of the "W" mine.

In the year 1929 in the "W" mine, as well as in all Poland, we see an increase of costs of production. In the year 1930 there follows an average further increase of costs in all Poland, and in the "W" mine already in this year begins a slight fall, which increases tremendously in the years 1931 and 1932, in the "W" mine as well as in Poland as a whole.

Costs are a mathematical function of production. In order to justify the oscillations of the costs of production of coal in the years 1928—1932, it is necessary first of all to determine the average amounts mined in these years. It is also necessary to ascertain the seasonal oscilations in mining during the course of each year, which periodically appear in certain months.

The costs of labour increase in the years 1929 and 1930 in relation to the preceding years. In the year 1931 they are lower than in the year 1930, and in the year 1932 we note their further decrease.

These costs together with the entire costs of production are above all dependent upon the quantity mined in the various years. The amount of the cost of labour, however, depends likewise upon 1) the number of labourers, 2) the efficiency of the labourer, 3) the intensity of work, 4) the pay of the labourers. Only a detailed examination of all of these factors gives us a sufficient justification for the oscillations of the costs of labour.

The increase of the cost of labour in the year 1929 may be connected with the decrease of efficiency as a result of the fact that the increase in the number of labourers was greater than the amount mined, and with increase in the labourers' wages unproportional to their efficiency. The further increase in the costs of labour in the year 1930 may be explained by the fact that to the fall in amount mined does not correspond a proportional fall in the number of labourers busy in the mines as bufforwork, preparation etc. This is proved by the large fall in efficiency, and by the increase in the number of holidays. At the same time the wages remain unchanged, and the average wage of the labourers increases as the increase of wages introduced in the course of the preceding year still hold for the whole of the year 1930. The fall in the cost of labour in the year 1931 may be associated with the increase in the amount mined and with the simultaneous reduction in the number of labourers,

which with practically the same number of holidays brings about an increase in efficiency. The further fall of costs in the year 1932 is explained by the decrease in the number of labourers, greater than the decrease in production. Mines are reorganized to decrease their costs, as a result efficiency increases. Other categories of the costs of production of coal in the mine "W" followed similar oscillations in the years 1928—1932.

Oscillations of profit are examined directly through a comparison of the oscillations of prices and the costs of production. From such a comparison we see that profits in the coal industry from the year 1928 to 1932 are constantly smaller.

Dr Stefan Bolland, *Studjum nad metodą statystycznego badania terytorjalnej jednorodności rynku. Czasopismo prawnicze 1935. (Study on the Statistical Method of Examining the Territorial Homogeneity of the Market. 74 pages. Cracow 1935).*

The author begins his paper with the assertion that between the pure Economic Theory and empirical economic reality there arise certain differences, which the author accredits to frictions phenomena. The author is of the opinion that by the examination, especially quantitative of these phenomena, by the bestowing on them of qualities permitting of measurement and comparison, and by the closer definition of the influence which they have on the course of "essential" phenomena from the point of view of theory, the science of economy would gain in definiteness. In his paper the author attempts to work out a method of this type of experiments on frictions phenomena in the limits of the dimensional shaping of prices.

Explaining his point of view in the introductory chapter, devoted to the discussion of methodological problems, the author passes to the analysis of the theoretical conception of market. After explaining his conception he comes to the conclusion that in reality we rarely meet with a dimensional correspondence on the market in the theoretical sense of the word, since the prices of merchandise (and services) show local differences greater than the transportation costs (of a given merchandise). Making use of the papers (statistical and descriptive-

statistical) of Mills, Englebrecht, Fabian, Jankowski, and others, and basing his observations on his own studies of prices in Poland, the author presents various phenomena in the course of price-changes in the frequency and amplitude of these changes, etc.

That condition in which the territorial system of prices corresponds in entirety to the theoretical market, the author terms a perfectly homogeneous market. As we have mentioned, the author is of the opinion that perfectly homogeneous markets, especially if it is a question of larger territories, practically do not exist. However, since particular territories are more or less remote from that ideal, the author wishes to be able to measure and express comparatively the degree of market homogeneity of various territories in relation to the same merchandise, and of various merchandise in relation to the same territories.

The comparison alone of the differences arising between prices in various localities and the costs of transport between those localities. In order to find out whether these localities are a homogeneous market, does not lead to satisfactory results. For this purpose the author proposes a method, whose conception forms the original part of the paper. The author, namely, reached the conclusion that in the case of the existence of a perfectly homogeneous market, the factor of the interlocal dispersion of prices (average deviation of the local prices from their aritmetical mean) is a fixed amount, (abstracting, however, from the long-period diminishing tendency which that factor shows, as well as from the changes called forth by the general trend of prices). For if the differences between the local prices correspond to the costs of transportation between given places, then the amount of these differences (in unchanged costs of transportation) is constant, independent of the changes of the amounts of the prices themselves.

Observing the interlocal dispersion of the prices of various merchandise in Poland (in the course of several years) and analyzing the results of the studies of Mills, the author has proved that in reality the factors of the interlocal dispersion of prices show a definite tendency to maintain a fixed level. The degree of this fixedness, as it seems, very different for different types of merchandise, shows, according to the author,

a greater or smaller homogeneity of the market. A completely fixed factor of interlocal dispersion in prices would belong only to a perfectly homogeneous market. The oscillations of the value of this factor shows the lack of a homogeneous market. Comparing the amount of these oscillations it is possible to discover which of the markets compared is more and which is less homogeneous.

Developing further problems of a technical statistical nature, the author introduces the "criterium of market homogeneity":

$$L_k = \frac{k_t}{m_t} \cdot 100$$

where k_t equals the mean or average deviation of a chronological series of the factors of the interlocal dispersion of prices and m_t the arithmetical mean of the local prices. This formula may be applied in several-year periods. But in longer periods of time, in the course of which a diminution trend might appear in interlocal dispersion, then it is advisable to eliminate that trend by applying the "criterium" in a somewhat modified form:

$$L_k = \frac{1}{n} \sum_{i=1}^{i=n} \left| \frac{\delta_i}{y_i} \right|$$

where δ_i designates the differences arising from the subtraction of the various factors of dispersion (t_i) from the present values of the trend (y_i) and n designates the number of years for which the factors (t_i) were calculated.

To illustrate this method, the author examines according to it the homogeneity of the territory of Poland as a market for various merchandise. The order of the markets taken under consideration, arranged according to their growing values L_k, and thus beginning with the most homogeneous market is presented as follows:

Beef	0·99	Soap . . .	4·67	Potatoes . . .	16·48
Oxen	2·21	Wheat flour	7·22	Printers' salaries	25·—
Coal	3·23	Eggs . . .	7·78	Masons salaries .	35·40

The market is the more perfectly homogeneous, the smaller the value of the "criterium" L_k. These values are useful, however, only in making comparisons. They show, that, for ins-

tance, Poland is a more homogeneous market for meat than for soap. Of course, the value of the "criterium" cannot fall below zero, whence values near zero show a comparatively considerable market homogeneity. We do not know, however, what is a perfectly homogeneous market, nor do we know how great is the extent of the scale on which we mark the values L_k and therefore we lack a perspective for the evaluation of these values.

Further the author attempts to separate certain areas from the territory of Poland, which form more homogeneous markets for certain merchandise. The object of the paper, however, is not to make detailed studies, but to discuss a method for that type of study.

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